

Video Digitization

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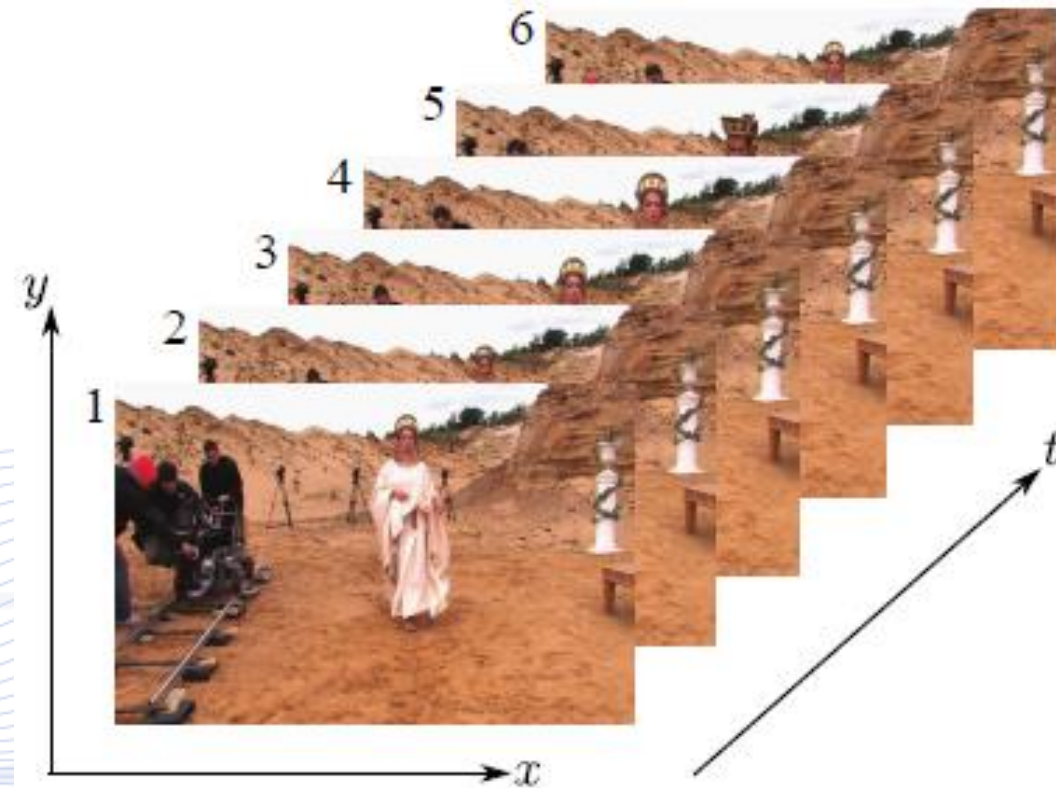
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Video Digitization

- **3D data types**
- Video sampling
- Progressive video sampling
- General video sampling grids
- General analog video reconstruction

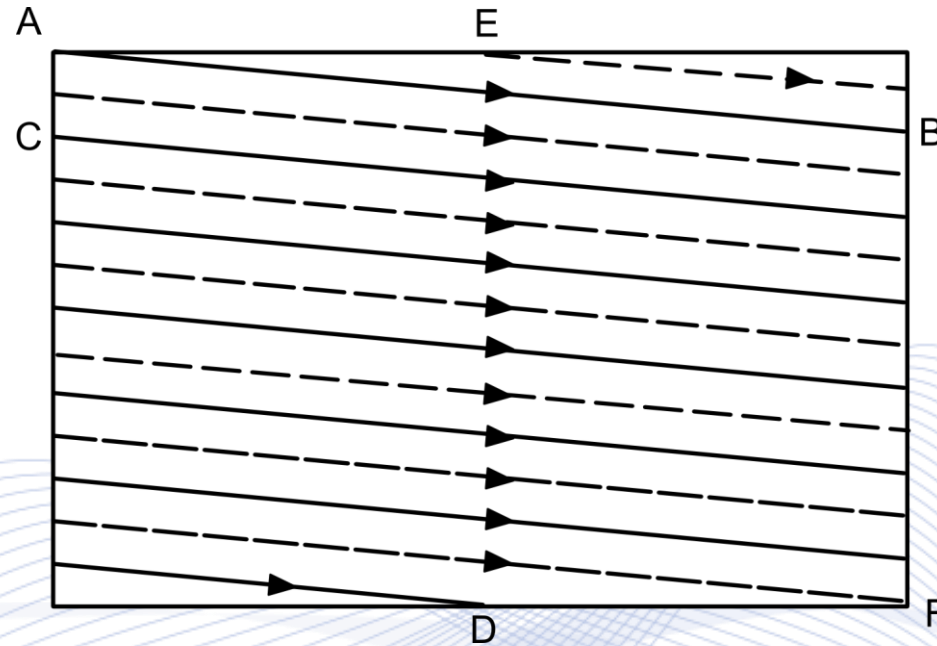
3D data types: video



3D data types: video

- **Moving images** are **spatiotemporal 3D signals** of the form:
 $f(x, y, t): \mathbb{R}^3 \rightarrow \mathbb{R}$, having:
 - domain \mathbb{R}^3 and codomain \mathbb{R} .
 - the time t coordinate has a different nature than the spatial coordinates x, y .
- **Video scanning**: creation of an 1D analog video signal, by sampling the time-varying images (luminance or RGB channels) along the vertical axis y and time t .

3D data types: video



Analog video scanning.

3D data types: video, cinema

- Analog video signal $f(x, j\Delta y, k\Delta t)$: $\mathbb{R} \times \mathbb{Z}^2 \rightarrow \mathbb{R}$.
 - discrete along y and t axes
 - continuous along x axis.
- Cinema moving images $f(x, y, k\Delta t)$: $\mathbb{R}^2 \times \mathbb{Z} \rightarrow \mathbb{R}$.
- Digital video signal $f(i\Delta x, j\Delta y, k\Delta t)$: $\mathbb{Z}^3 \rightarrow \mathbb{R}$.

3D data types: video, cinema



- **Spatial sampling intervals** $\Delta x, \Delta y$ define **image resolution**: the smaller they are, the smaller the pixel size is.
 - **HDTV image resolution** 1080p: 1080×1920 pixels.
- **Temporal sampling interval** Δt defines the **video frame rate** in **frames per second (fps)**.
 - Typical fps: 25 (Europe), 30 (USA).

3D data types: volumetric images

- **3D volumetric images:** 3D signals of the form:

$$f(x, y, z): \mathbb{R}^3 \rightarrow \mathbb{R}.$$

- Discrete versions (defined on a Euclidean grid \mathbb{Z}^3):

$$f(n_1, n_2, n_3): \mathbb{Z}^3 \rightarrow \mathbb{R}.$$

- $x = n_1 \Delta x$, $y = n_2 \Delta y$, $z = n_3 \Delta z$.
- $\Delta x, \Delta y, \Delta z$: **spatial sampling intervals** define 3D image resolution.
- Each **voxel** is a real number.

3D data types : volumetric images



Image gallery of a 3D volume.

3D data types: multispectral images

- **Multispectral/multichannel** (n -channel) images have the form: $f(x, y): \mathbb{R}^2 \rightarrow \mathbb{R}^n$.

- Color images ($n = 3$):

$$f(x, y) = [f_R(x, y), f_G(x, y), f_B(x, y)]^T: \mathbb{R}^2 \rightarrow \mathbb{R}^3.$$

- Digital color images (assigning 8 bits per color channel to each pixel): $f(n_1, n_2): \mathbb{Z}^2 \rightarrow \{0, \dots, 255\}^3$.

- They are also 3D images $f(n_1, n_2, i)$, $i = 1, 2, 3$.

- **Hyperspectral images** (3D images): $f(x, y, \lambda): \mathbb{R}^3 \rightarrow \mathbb{R}$.

- λ wavelength.

3D data types: memory issues



- 3D volumetric images have relatively small number of 3D image slices.
- Video has very large number of video frames:
 - 1 hour video: $60 \times 60 \times 50 = 180,000$ video frames.
- High frame rates: 25/50 up to 1000 Hz.
 - Use in recording high-speed phenomena.
- ***Video buffering:***
 - Only few video frames are stored in RAM.
 - Buffer update in live streams.

3D data types

- **Multiview images:** images of an object or set, taken from different view points, typically using different cameras.
 - **Stereo images:** a special case, employing only two cameras (left and right).
 - Video synchronization issues.

3D data types

- They both carry only implicit geometrical information about the visualized 3D object.
 - They are ***not*** 3D data.
 - 3D object geometry can be derived using stereo or multiview 3D geometry reconstruction techniques.

3D data types



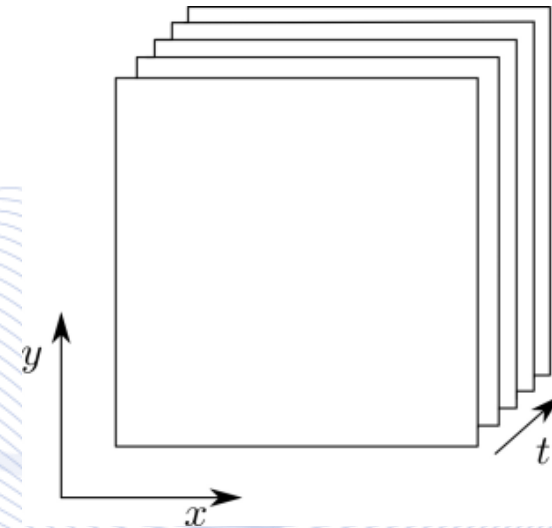
Multiview video captured by synchronized video-cameras.

Video Digitization

- 3D data types
- **Video sampling**
- Progressive video sampling
- General video sampling grids
- General analog video reconstruction

Video sampling

- Analog video signal is an image sequence $f(x, y, t)$.
 - x : horizontal coordinate,
 - y : vertical coordinate,
 - t : time coordinate.
- Digital video signal is obtained by spatiotemporal sampling of analog video along its coordinates x, y, t .



Spatiotemporal video signal.

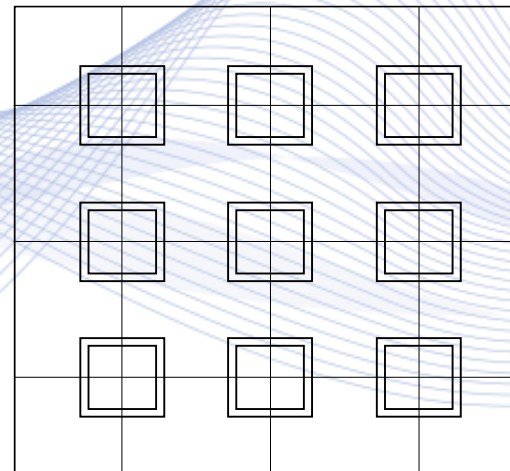
Video sampling

- **Analog video scanning:**
 - 3D luminance signal is scanned along the horizontal axis x and then over time t .
 - During analog video transmission, 1D signals $f(x, j\Delta y, \kappa\Delta t)$ are concatenated to form an 1D analog signal as a function of time that can be broadcasted.
 - The analog video signal is discrete along axes y, t and continuous along axis x .

Video sampling

- Digital video can be obtained:
 - by sampling analog video along the horizontal scan lines,
 - by using the discrete 2D sampling grid inherent in photoelectric sensors, e.g., in CCD chips.

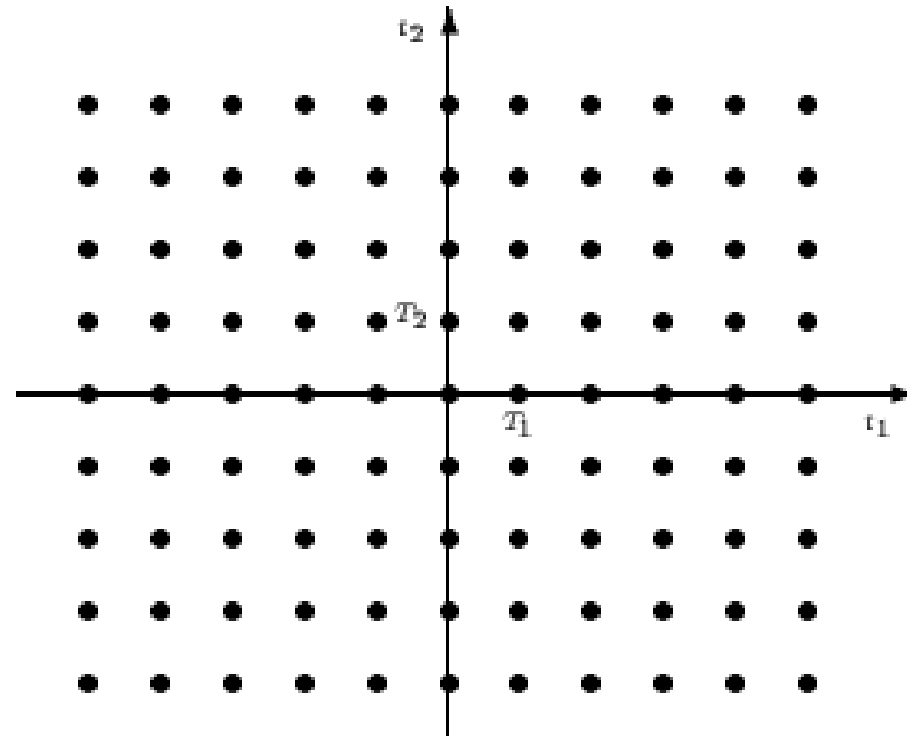
CCD chip.



Video Digitization

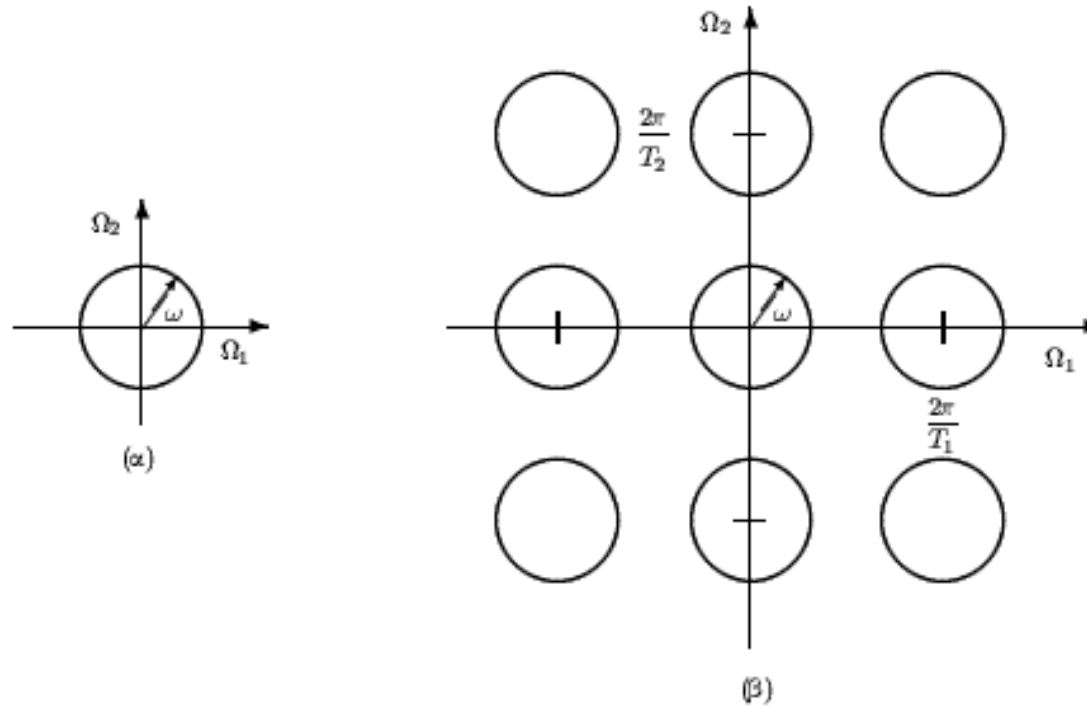
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2D image sampling



Rectangular image sampling grid.

2D sampled image spectrum

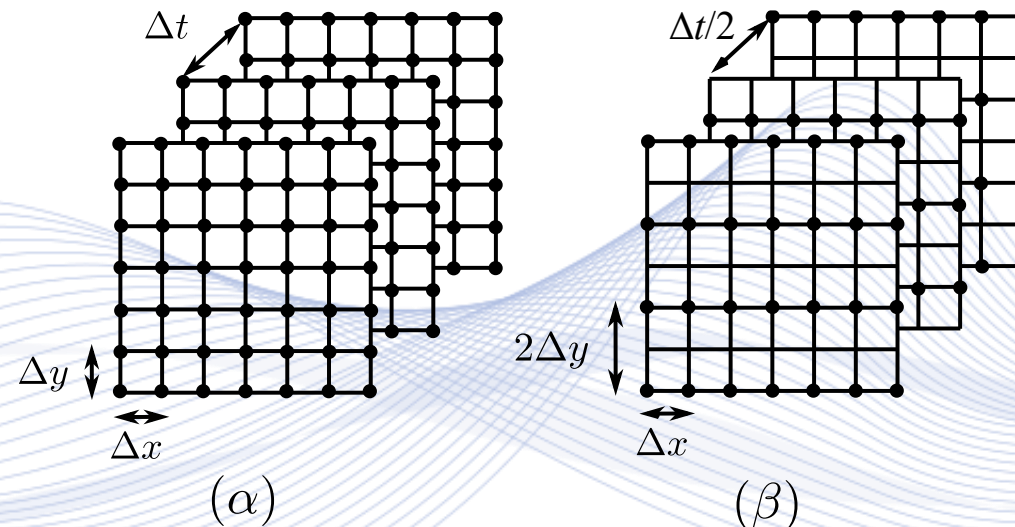


Fourier Transform of: a) Analog image; b) Discrete image.

Progressive video sampling

Progressive sampling:

- uniform spatiotemporal analog video sampling along x, y, t .



Sampling grids for: a) Progressive; b) 2:1 interlaced video.

Progressive video spectrum

Discrete (sampled) video $f(n_1, n_2, n_t)$:

$$f(n_1, n_2, n_t) = f_a(n_1\Delta x, n_2\Delta y, n_t\Delta t).$$

- $f_a(x, y, t)$: analog spatiotemporal video signal.
- $\Delta x, \Delta y, \Delta t$: sampling intervals along the horizontal/vertical spatial axis x, y and time axis t .

Progressive video sampling



Forward and inverse **3D Fourier transform** of analog video:

$$F_a(\Omega_x, \Omega_y, \Omega_t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_a(x, y, t) e^{-i\Omega_x x - i\Omega_y y - i\Omega_t t} dx dy dt,$$

$$f_a(x, y, t) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F_a(\Omega_x, \Omega_y, \Omega_t) e^{i\Omega_x x + i\Omega_y y + i\Omega_t t} d\Omega_x d\Omega_y d\Omega_t.$$

- $\Omega_x = 2\pi F_x$, $\Omega_y = 2\pi F_y$, $\Omega_t = 2\pi F_t$: **spatiotemporal frequencies**.
- They describe video content variations over space/time.

Progressive video spectrum

- Smooth video content is represented by low frequency content:
 - **DC term:** $\Omega^T = [\Omega_x, \Omega_y, \Omega_t]^T = [0,0,0]^T$ dominates.
- Image details and video content changes along x, y axis lead to high spatial frequency Ω_x, Ω_y content:
 - Edges, lines, textured regions.
- Fast video content changes over time lead to high Ω_t frequency content:
 - e.g., in the case of fast moving objects.

Progressive video spectrum

3D discrete spatiotemporal Fourier transform:

$$F(\omega_x, \omega_y, \omega_t) = \sum_{n_1} \sum_{n_2} \sum_{n_3} f(n_1, n_2, n_3) e^{-i(\omega_x n_1 + \omega_y n_2 + \omega_t n_3)} \quad (4)$$

and:

$$f(n_1, n_2, n_3) = \left(\frac{1}{2\pi}\right)^3 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} F(\omega_x, \omega_y, \omega_t) e^{i(\omega_x n_1 + \omega_y n_2 + \omega_t n_3)} d\omega_x d\omega_y d\omega_t \quad (5)$$

- $\omega_x = \Omega_x \Delta x$, $\omega_y = \Omega_y \Delta y$, $\omega_t = \Omega_t \Delta t$.
- they are defined on a unit circle: $\omega_x, \omega_y, \omega_t \in [-\pi, \pi]$.

Progressive video spectrum

- The triple integral can be divided into a triple sum of integrals:

$$f(n_1, n_2, n_t) = \frac{1}{(2\pi)^3} \sum_{k_x} \sum_{k_y} \sum_{k_t} \int \int \int_{RP(k_x, k_y, k_t)} \frac{1}{\Delta x \Delta y \Delta t} F_a\left(\frac{\omega_x}{\Delta x}, \frac{\omega_y}{\Delta y}, \frac{\omega_t}{\Delta t}\right) e^{i\omega_x n_1 + i\omega_y n_2 + i\omega_t n_t} d\omega_x d\omega_y d\omega_t.$$

- Each of them is defined on the shifted parallelepipeds

$RP(k_x, k_y, k_t)$:

$$\begin{aligned} -\pi + 2\pi k_x &\leq \omega_x \leq \pi + 2\pi k_x, \\ -\pi + 2\pi k_y &\leq \omega_y \leq \pi + 2\pi k_y, \\ -\pi + 2\pi k_t &\leq \omega_t \leq \pi + 2\pi k_t. \end{aligned}$$

Progressive video spectrum

- By interchanging summations with integrals and changing variables, the 3D discrete spatiotemporal Fourier transform is given by:

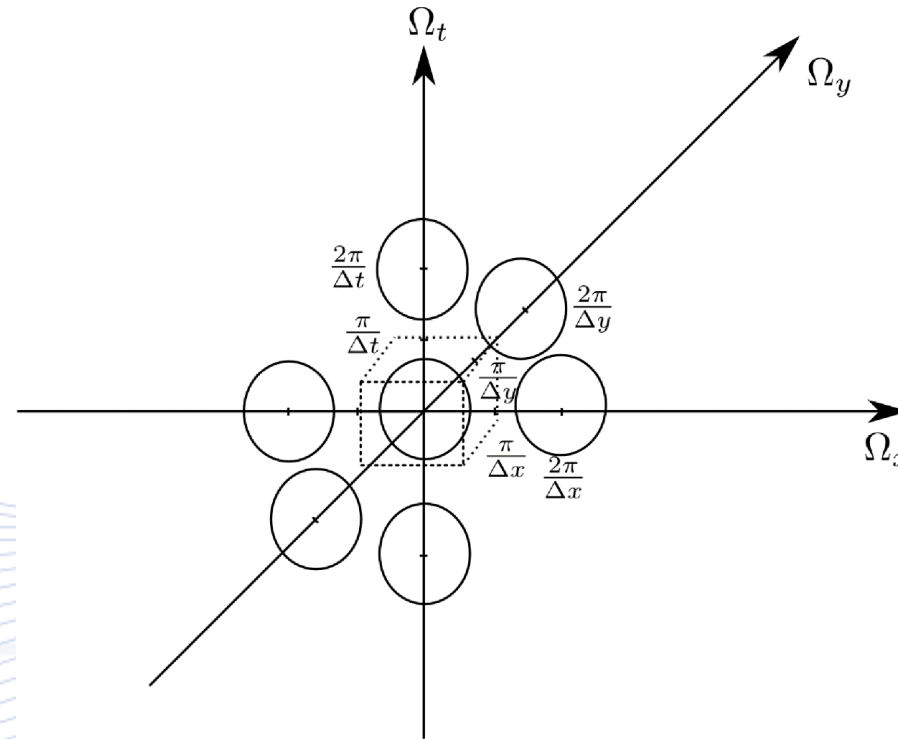
$$F(\omega_x, \omega_y, \omega_t) = \frac{1}{\Delta x \Delta y \Delta t} \sum_{k_x} \sum_{k_y} \sum_{k_t} F_a\left(\frac{\omega_x - 2\pi k_x}{\Delta x}, \frac{\omega_y - 2\pi k_y}{\Delta y}, \frac{\omega_t - 2\pi k_t}{\Delta t}\right),$$

or

$$F(\Omega_x \Delta x, \Omega_y \Delta y, \Omega_t \Delta t) = \frac{1}{\Delta x \Delta y \Delta t} \sum_{k_x} \sum_{k_y} \sum_{k_t} F_a\left(\Omega_x - \frac{2\pi k_x}{\Delta x}, \Omega_y - \frac{2\pi k_y}{\Delta y}, \Omega_t - \frac{2\pi k_t}{\Delta t}\right).$$

- Periodic translation of the 3D Fourier transform $F_a(\Omega_x, \Omega_y, \Omega_t)$ of the analog video $f_a(x, y, t)$.

Progressive video spectrum



3D periodic translation of the analog video spectrum.

Progressive video spectrum

If the sampling intervals Δx , Δy , Δt are sufficiently small, so that:

$$F(\Omega_x \Delta x, \Omega_y \Delta y, \Omega_t \Delta t) = 0, \quad |\Omega_x| \geq \frac{\pi}{\Delta x}, \quad |\Omega_y| \geq \frac{\pi}{\Delta y}, \quad |\Omega_t| \geq \frac{\pi}{\Delta t}.$$

the discrete video spectrum is given by the periodic translation of the fundamental period:

$$F(\Omega_x \Delta x, \Omega_y \Delta y, \Omega_t \Delta t) = \frac{1}{\Delta x \Delta y \Delta t} F_a(\Omega_x, \Omega_y, \Omega_t),$$

$$|\Omega_x| \leq \frac{\pi}{\Delta x}, \quad |\Omega_y| \leq \frac{\pi}{\Delta y}, \quad |\Omega_t| \leq \frac{\pi}{\Delta t}.$$

Reconstruction of analog video

Analog video can be reconstructed from the discrete video:

$$f_a(x, y, t) = \sum_{n_1} \sum_{n_2} \sum_{n_t} f(n_1, n_2, n_t) \frac{\sin \frac{\pi}{\Delta x}(x - n_1 \Delta x)}{\frac{\pi}{\Delta x}(x - n_1 \Delta x)} \frac{\sin \frac{\pi}{\Delta y}(y - n_2 \Delta y)}{\frac{\pi}{\Delta y}(y - n_2 \Delta y)} \frac{\sin \frac{\pi}{\Delta t}(t - n_t \Delta t)}{\frac{\pi}{\Delta t}(t - n_t \Delta t)}.$$

- Mathematical modeling of digital video visualization/projection.
- In reality, much simpler approaches are used:
 - Zero-order interpolation
 - Square image blobs can be visible.

Reconstruction of analog video

- If $\Delta x, \Delta y, \Delta t$ are sufficiently small, analog video can be precisely reconstructed.
- If the video signal is not low-pass:
 - too many image details and/or very strong motion.
- or the sampling intervals $\Delta x, \Delta y, \Delta t$ are not sufficiently small, ***spectrum aliasing*** occurs.

Reconstruction of analog video

- Spectrum content overlaps in high frequency areas:

$$|\Omega_x \Delta x| \leq \pi, \quad |\Omega_y \Delta y| \leq \pi, \quad |\Omega_t \Delta t| \leq \pi.$$

- High frequency video content is smoothed out (details and strong motion).

Reconstruction of analog video

- **Nyquist criterion:**

$$\Delta x \leq \frac{\pi}{\Omega_{xmax}}, \quad \Delta y \leq \frac{\pi}{\Omega_{ymax}}, \quad \Delta t \leq \frac{\pi}{\Omega_{tmax}},$$

to avoid aliasing problems.

- **Sampling frequencies** $F_{sx} = \frac{1}{\Delta x}$, $F_{sy} = \frac{1}{\Delta y}$, $F_{st} = \frac{1}{\Delta t}$ must satisfy:

$$F_{sx} \geq 2F_{xmax}, \quad F_{sy} \geq 2F_{ymax}, \quad F_{st} \geq 2F_{tmax}.$$

- F_{xmax} , F_{ymax} , F_{tmax} : maximal spatial and temporal video frequencies.

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General video sampling grids

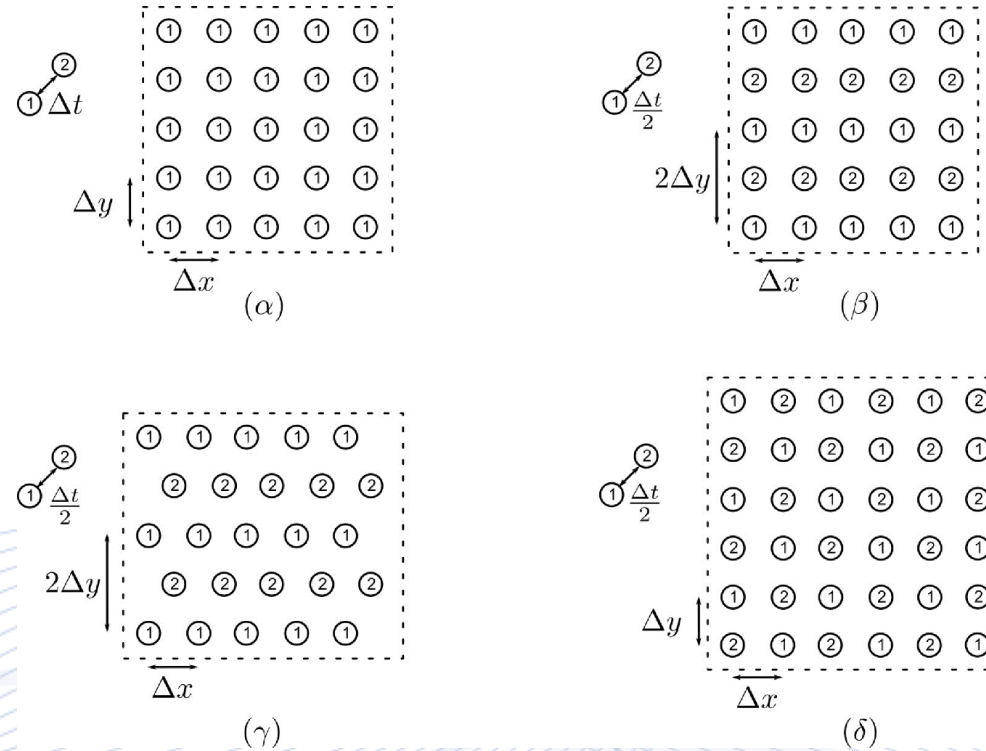


- Multidimensional signal $f(\mathbf{x})$ over \mathbb{R}^K .
- Video: special multidimensional signal for $K = 3$.
- **3D sampling grid**: mathematical structure describing video sampling (also called **lattice**):

$$\Lambda = \left\{ \mathbf{x} \in \mathbb{R}^K \mid \mathbf{x} = \sum_{k=1}^K n_k \mathbf{v}_k, \quad \forall n_k \in \mathbb{Z} \right\}.$$

- $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_K$: Linearly independent basis vectors on \mathbb{R}^K .
- $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_K]$: **sampling matrix**.

General video sampling grids



a) Rectangular sampling grid; b) Vertically aligned interlaced grid; c) Quincunx grid; d) Orthorhombic grid.

General video sampling grids

- Any grid point can be represented by vector:

$$\mathbf{n} = [n_1, n_2, \dots, n_K] \in \mathbb{Z}^K.$$

- Actual pixel position in \mathbb{R}^K : $\mathbf{x} = \mathbf{V}\mathbf{n}$.
- Sampling matrix or basis vectors fully define a sampling grid.
- They are not unique:
- \mathbf{V} and $\mathbf{V}' = \mathbf{E}\mathbf{V}$ define the same sampling grid Λ , if:
- \mathbf{E} : integer matrix and $\det(\mathbf{E}) = \pm 1$.

General video sampling grids

- Progressive sampling grid matrix:

$$\mathbf{V} = \begin{bmatrix} \Delta x & 0 & 0 \\ 0 & \Delta y & 0 \\ 0 & 0 & \Delta t \end{bmatrix}.$$

- Vertically aligned interlaced grid matrix :

$$\mathbf{V} = \begin{bmatrix} \Delta x & 0 & 0 \\ 0 & 2\Delta y & \Delta y \\ 0 & 0 & \Delta t/2 \end{bmatrix}.$$

General video sampling grids

- Quincunx grid matrix:

$$\mathbf{V} = \begin{bmatrix} \Delta x & 0 & \Delta x/2 \\ 0 & 2\Delta y & \Delta y \\ 0 & 0 & \Delta t/2 \end{bmatrix}.$$

- Orthorhombic grid matrix:

$$\mathbf{V} = \begin{bmatrix} 2\Delta x & \Delta x & \Delta x \\ 0 & \Delta y & 0 \\ 0 & 0 & \Delta t/2 \end{bmatrix}.$$

General video sampling grids



- Each grid has a unit cell, whose definition is not unique.
- **Unit grid cell (Voronoi cell)** $\mathcal{V}(\Lambda)$: the set of points on \mathbb{R}^K that are closest to $\mathbf{0}$, then to any other grid point:

$$\mathcal{V}(\Lambda) = \{\mathbf{x} \in \mathbb{R}^K \mid d(\mathbf{x}, \mathbf{0}) \leq d(\mathbf{x}, \mathbf{p}), \quad \forall \mathbf{p} \in \Lambda\}.$$

- $d(\mathbf{x}, \mathbf{p})$: Euclidean distance of points $\mathbf{x}, \mathbf{p} \in \mathbb{R}^K$.

General video sampling grids



- **Grid sampling density:** it is the number of grid nodes that exist in the unit volume in \mathbb{R}^K .
- It equals the inverse of the unit cell volume:

$$D(\Lambda) = \frac{1}{|\det \mathbf{V}|}.$$

General video sampling grids



- **Inverse grid** Λ^* is defined by the grid sampling matrix \mathbf{U} :

$$\mathbf{V}^T \mathbf{U} = 2\pi \mathbf{I}.$$

- **Sampling grid density** of the inverse grid Λ^* :

$$D(\Lambda^*) = \frac{1}{(2\pi)^3 D(\Lambda)} = \frac{|\det \mathbf{V}|}{(2\pi)^3}.$$

- It is inversely proportional to that of the sampling lattice Λ .

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General video sampling

- $f_a(\mathbf{x}_t)$, $\mathbf{x}_t = [x, y, t]^T \in \mathbb{R}^3$: analog video.
- Video sampling using sampling matrix \mathbf{V} :

$$f(\mathbf{n}) = f_a(\mathbf{x}_t) \sum_{\mathbf{n} \in \mathbb{Z}^3} \delta(\mathbf{x}_t - \mathbf{V}\mathbf{n}).$$

- $\mathbf{n} = [n_1, n_2, n_t]^T \in \mathbb{Z}^3$.

General video sampling

- Discrete spatiotemporal Fourier transform:

$$F(\boldsymbol{\omega}) = \sum_{\mathbf{n} \in \mathbb{Z}^3} f(\mathbf{n}) e^{i\boldsymbol{\Omega}^T \mathbf{V} \mathbf{n}},$$

- $\boldsymbol{\omega} = \mathbf{V}^T \boldsymbol{\Omega}$.
- $\mathbf{U} = 2\pi(\mathbf{V}^T)^{-1}$: sampling matrix the inverse grid Λ^* .
- If $\mathbf{n}^T \mathbf{k} = m \in \mathbb{Z}$, then $F(\boldsymbol{\omega} + 2\pi \mathbf{k}) = F(\boldsymbol{\omega})$.
- The Fourier transform of a sample video on a lattice Λ is periodic on lattice Λ^* on \mathbb{R}^3 .

General video sampling

- Discrete video spectrum:

$$F(\boldsymbol{\omega}) = \frac{1}{|\det \mathbf{V}|} \sum_{\mathbf{k} \in \mathbb{Z}^3} F_a\left(\frac{1}{2\pi} \mathbf{U}(\boldsymbol{\omega} - 2\pi \mathbf{k})\right).$$

- Sampling grid Λ ,
- Periodic translation of the analog video spectrum on the inverse grid Λ^* .

General video sampling

Spectrum aliasing:

- The denser the sampling grid Λ is, the more sparse is the inverse Λ^* grid.
- If the support area of the continuous video signal spectrum is bigger than the unit cell P of the grid Λ^* , the nearby areas of the translated spectrum version overlap.
- Alias reduction: use a denser sampling grid.

General video sampling

- Progressive video frequency periodicity grid Λ^* :

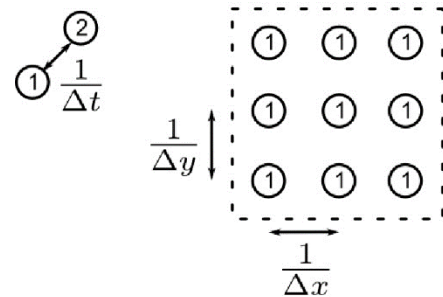
$$\mathbf{U} = 2\pi(\mathbf{V}^T)^{-1} = \begin{bmatrix} \frac{2\pi}{\Delta x} & 0 & 0 \\ 0 & \frac{2\pi}{\Delta y} & 0 \\ 0 & 0 & \frac{2\pi}{\Delta t} \end{bmatrix},$$

- 2:1 interlaced frequency periodicity grid Λ^* :

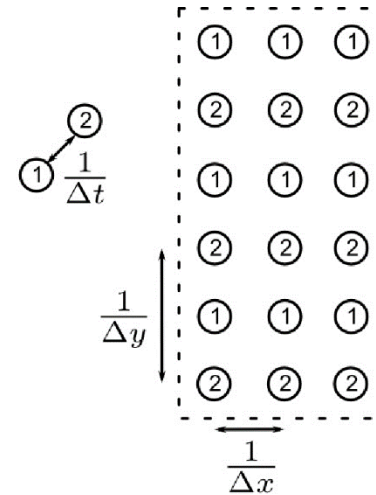
$$\mathbf{U} = 2\pi(\mathbf{V}^T)^{-1} = \begin{bmatrix} \frac{2\pi}{\Delta x} & 0 & 0 \\ 0 & \frac{\pi}{\Delta y} & 0 \\ 0 & -\frac{2\pi}{\Delta t} & \frac{4\pi}{\Delta t} \end{bmatrix}.$$

General video sampling

Frequency periodicity grids Λ^* :



(α)



(β)

Inverse sampling grid Λ^* for: a) Progressive video;
b) 2:1 interlaced video.

General analog video reconstruction



Analog video reconstruction:

- Original analog signal $f_a(\mathbf{x}_t)$ recovery from $f(\mathbf{n})$:
 - the support area of the Fourier transform $F_a(\Omega_x, \Omega_y, \Omega_t)$ should be a subset of the unit cell of the inverse grid Λ^* .
- Perfect reconstruction by an ideal low-pass filter:
 - Its passband should be the unit cell \mathcal{P} and the Λ^* grid:

$$F_a(\boldsymbol{\Omega}) = \begin{cases} |\det \mathbf{V}| F(\mathbf{V}^T \boldsymbol{\Omega}), & \boldsymbol{\Omega} \in \mathcal{P} \\ 0, & \text{elsewhere.} \end{cases}$$

General analog video reconstruction

- Analog video reconstruction:

$$f_a(\mathbf{x}_t) = \sum_{\mathbf{n} \in \mathbb{Z}^3} f(\mathbf{n}) h(\mathbf{x}_t - \mathbf{V}\mathbf{n}).$$

- Impulse response of the ideal spatiotemporal low-pass interpolation filter:

$$h(\mathbf{x}_t) = \frac{|\det \mathbf{V}|}{(2\pi)^3} \int_{\mathbf{P}} e^{i\boldsymbol{\Omega}^T \mathbf{x}_t} d\boldsymbol{\Omega}.$$

General analog video reconstruction

- For progressive video sampling:
 - the impulse response of the ideal video interpolation filter is the triple sinc function:

$$h(\mathbf{x}_t) = \frac{\sin(\frac{\pi}{\Delta x}x)}{\frac{\pi}{\Delta x}x} \frac{\sin(\frac{\pi}{\Delta y}y)}{\frac{\pi}{\Delta y}y} \frac{\sin(\frac{\pi}{\Delta t}t)}{\frac{\pi}{\Delta t}t}.$$

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Q & A

Thank you very much for your attention!

**More material in
<http://icarus.csd.auth.gr/cvml-web-lecture-series/>**

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