

Color image histogram equalization by absolute discounting back-off

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Abstract

A novel color image histogram equalization approach is proposed that exploits the correlation between color components and it is enhanced by a multi-level smoothing technique borrowed from statistical language engineering. Multi-level smoothing aims at dealing efficiently with the problem of unseen color values, either considered independently or in combination with others. It is applied here to the HSI color space for the probability of intensity and the probability of saturation given the intensity, while the hue is left unchanged. Moreover, the proposed approach is extended by an empirical technique, which is based on a hue preserving non-linear transformation, in order to eliminate the gamut problem. This is the second method proposed in the paper. The equalized images by the two methods are compared to those produced by other well-known methods. The better quality of the images equalized by the proposed methods is judged in terms of their visual appeal and objective figures of merit, such as the entropy and the Kullback–Leibler divergence estimates between the resulting color histogram and the multivariate uniform probability density function.

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1. Introduction

Image enhancement aims at improving images from the human visual perspective. Image features such as edges, boundaries, and contrast are sharpened in a way that their dynamic range is increased without any change in the information content inherent in the data [1]. For this purpose, several techniques have been developed. Among others are contrast manipulation, noise reduction, edge crispening and sharpening, filtering, pseudocoloring, image interpolation and magnification [1].

Contrast manipulation techniques can be classified as either *global* or *adaptive*. *Global* techniques apply a transformation to all image pixels, while *adaptive* techniques use an input–output transformation that varies adaptively with the local image characteristics. The more common

global techniques are linear contrast stretch, histogram equalization, and multichannel filtering. The most common adaptive techniques are adaptive histogram equalization (AHE) and contrast-limited adaptive histogram equalization (CLAHE) [2,3]. AHE applies varying gray-scale transformations locally to every small image region, thus requiring the determination of the region size. CLAHE improves the just described technique by limiting the local contrast-gain. Two drawbacks of the latter method have been identified namely the unavoidable enhancement of noise in smooth regions and the image-dependent selection of the contrast-gain limit [4].

This paper is focused on global techniques with emphasis to color images. More precisely, the notion of unigram and bigram probabilities together with probability smoothing, borrowed from statistical language modeling, is applied to color histogram equalization in order to jointly equalize the two components of the HSI color space, namely the saturation and the intensity. The histogram equalization approach is partially built on that proposed

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in Pitas and Kiniklis [5], but it is extended with smoothing the necessary probabilities in order to counteract the effect of unseen color component combinations, which stems from the dimensionality of the color space and the often limited number of colors present in an image. Additionally, a second method is developed in an effort to eliminate the gamut problem by exploiting the transformations proposed in Naik and Murthy [6]. The performance of the proposed methods is compared to that of the methods proposed by Pitas and Kiniklis [5,7] as well as the separate equalization of each color component. The comparison is conducted using not only subjective measures (i.e., how visually appealing the equalized images are), but also objective figures of merit, such as the entropy and the Kullback–Leibler divergence between the resulted color histogram and the corresponding multivariate uniform probability density function.

The outline of the paper is as follows. In Section 2, the color image histogram equalization methods are briefly presented. In Section 3, the baseline histogram equalization approaches and the novel algorithms, proposed in this paper, are described. Experimental results are demonstrated in Section 4, and finally, conclusions are drawn in Section 5. A brief description of the RGB and HSI color spaces is given in Appendix A.

2. Related works

Histogram equalization is the simplest and most commonly used technique to enhance gray-level images. It assumes that the pixel gray levels are independent identically distributed random variables (rvs) and the image is a realization of an ergodic random field. As a consequence, an image is considered to be more informative, when its histogram resembles the uniform distribution. From this point of view, grayscale histogram equalization exploits the theory of functions of one rv that suggests using the cumulative distribution function (CDF) of pixel intensity in order to transform the pixel intensity to a uniformly distributed rv. However, due to the discrete nature of digital images, the histogram of the equalized image can be made approximately uniform.

Histogram equalization becomes a tedious task when dealing with color images due to the vectorial nature of

color. Each color pixel is represented by a vector with as many components as the color components in a proper color space (i.e., the three components Red, Green, and Blue in the RGB space). The complexity of the problem lies also in the correlation between the color components as well as the color perception by humans. The methods described in this paper either review the efforts to alleviate one or both these problems or revisit them in order to improve their performance.

Historically, the first and the most straightforward extension of histogram equalization to color images is the application of gray-scale histogram equalization separately to the different color bands of the color image, ignoring inter-component correlation. Some efforts were also focused in spreading the histogram along the principal component axes of the original image [8], or spreading repeatedly the three two-dimensional histograms [9].

The systematic research efforts that followed gave rise to two main algorithm classes. The first class comprises algorithms that work on the RGB space either using the 3-D histogram or an 1-D histogram of the color image. The second class is formed by algorithms, which operate in non-linear color spaces, such as the HSI (hue, saturation, and intensity) or the C-Y spaces, that are applied to one or two color components. The algorithms for each class are outlined in Tables 1 and 2 and described next.

In the RGB space, the more representative approaches are *3-D histogram equalization*, *histogram explosion*, and *histogram decimation*. *3-D histogram equalization* proposed by Trahanias and Venetsanopoulos [7], attempts the extension of cumulative histogram to higher dimensions by means of a uniform 3-D histogram specification in the RGB color cube. The 3-D CDF of the original image color is compared to the ideal uniform CDF, as is further explained in Section 3.2.

Histogram explosion, which was initially proposed by Mlsna and Rodriguez [10], aims to exploit the full 3-D RGB gamut. The algorithm selects an operating point preferably on the diagonal of the RGB cube (i.e. the gray line) in order to prevent hue changes and draws rays that emanate from the operating point, pass through the color points present in the image, and proceed up to the RGB cube facets. An 1-D histogram is then constructed along each ray by interpolating the 3-D histogram data between

Table 1
Color histogram equalization techniques in the RGB color space

Reference	Description
[7]	3-D histogram specification with the output histogram being uniform
[10]	Histogram Explosion: histogram equalization of the one-dimensional histograms constructed by interpolating the 3-D histogram data
[12]	Histogram Decimation: uniformly scatter the color points over the full 3-D gamut in an iterative procedure
[13]	The cells of a mesh, initially deformed to fit the original histogram, is linearly mapped to the cells of a uniform mesh
[15]	The image histogram is approximated by an isotropic Gaussian mixture and is fitted to a uniform distribution via least-squares error minimization (applicable to any dimensions)
[16]	Histogram equalization by exploiting the histogram utilization, which depends on the cumulative histograms of each color component given the other two color components, and their average occupancy
[14]	Histogram equalization treated as a color transfer problem, by estimating a transformation that maps a N -dimensional distribution to a uniform one

Table 2
Color histogram equalization techniques in color spaces other than the RGB

Reference	Color space	Description
[11]	CIE-LUV	Histogram Explosion (same as [10])
[21]	CIE	Grayscale histogram equalization of the achromatic channel and image warping of the chromatic channel
[19]	C-Y	Equalization of the saturation component within partitions of the color space, defined by the range of the hue and luminance components, and the luminance component over the whole image
[5]	HSI	Joint equalization of intensity and saturation by exploiting the geometric concepts between the HSI and RGB spaces
[17,18]	HSI	Histogram equalization of saturation component within each of 96 hue regions
[20]	HSI	Extension of [19] in HSI

the operating point and the point in the color space boundary. By equalizing the 1-D histogram, a new color value for the original color point is determined. The color points are thus almost uniformly spread in the color space. In another variant of the method, the colors are represented in the CIE-LUV space [2,11].

Histogram decimation uses an iterative algorithm to uniformly scatter the color points over the full 3-D gamut [12]. The algorithm is initialized with the entire 3-D color space as the current space and proceeds iteratively by applying two steps. In the first step, all color points within the current space are shifted such that their average coincides with the geometric center of the space. In the second step, the current color space is divided into eight equally sized subspaces. Each newly created color subspace is set as the current space for the next iteration. The algorithm stops, when the subspace size reaches its minimum value.

In the approach of Pichon et al. [13], a mesh is initially deformed to fit the original histogram in the RGB color space. The mesh is then used for the definition of a piecewise linear deformation of the color space by linearly mapping all its cells to the corresponding cells of a uniform mesh. Thus, the histogram of the resulting equalized image is always almost uniform, but the equalized image itself suffers from hue-relating artifacts. This fact makes the method particularly interesting for pseudo-color scientific visualization.

In Pitie et al. [14], the color transfer problem is addressed and a new method for estimating a transformation that maps a N -dimensional distribution to another is presented. The proposed method can be used for histogram equalization, when the target probability distribution is uniform. The algorithm operates iteratively in three steps. In the first step, the RGB space coordinate system is changed by rotating the source and the target samples using a proper rotation matrix. In the second step, all the samples are projected on the three axes in order to obtain the marginal distributions both for the rotated source and target samples. In the third step, an 1-D transformation that matches the source marginals into the target ones is found for each axis and the transformed samples are rotated back in the original RGB space. The algorithm stops when convergence on all marginals for every possible rotation is achieved.

A more recent method extends the grayscale histogram equalization to color images formulating the problem as

a nonlinear optimization problem with bound constraints [15]. The histogram of a given image in the RGB color space is approximated by an isotropic Gaussian mixture and its least squared error from the target uniform distribution is minimized.

In Forrest [16], a technique for measuring histogram utilization and performing histogram equalization to any number of dimensions has been developed. The histogram utilization measure is based on the fact that chi-squared measures from independent data can be summed to give an overall measure. Thus, the measure depends on the average histogram occupancy for the linear histograms of a color component (e.g. R) at a certain point lying on the plane of the other two color components (e.g. G and B) and the corresponding linear cumulative histograms. Histogram equalization is then achieved by exploiting the histogram utilization measure that generates information about how the histogram should be changed. In this way, the resulting histogram is effectively spread without large changes of contrast or color balance.

All the aforementioned approaches work in the RGB space. In general, they are computationally intensive and their major drawback is the modification of color hue. The latter fact leads to unpleasant color artifacts for the human observer. To alleviate hue modification, the second class of algorithms conduct equalization in the HSI space by modifying only the intensity, or both the intensity and saturation, leaving the hue unchanged.

A method that attempts to jointly equalize intensity and saturation was proposed by Pitas and Kiniklis [5]. To avoid unnatural colors after equalization, the method takes into account geometric concepts between the HSI space and the RGB space. To make this paper self-contained, the method is briefly reviewed in Section 3.3.

Another histogram equalization method is based on the intensity and saturation components [17,18]. In this method, the RGB color space is first transformed into an HSI triangle, that is divided into 96 hue regions. The histogram equalization is applied to the saturation within each hue region. The drawbacks of the algorithm lie in the determination of the maximum possible saturation value to be used for each transformation and the fact that the intensity component is neglected. The method was improved in Weeks et al. [19] by including both the saturation and the intensity component in the equalization process conducted

in the color difference (C-Y) space. It was found that the transformation yields unrealizable RGB colors in the equalized image. The algorithm partitions the entire C-Y color space, that is transformed into HSI, in $n \times k$ subspaces, where n and k are the number of partitions in hue and intensity components, respectively. Saturation is then equalized once within the maximum realizable saturation of each subspace. Next, the intensity component is equalized by considering the whole image. The method proposed by Weeks et al. [19] was further applied to the HSI color space by Duan and Qiu [20].

In Luccchese et al. [21], the achromatic channel of a color image was equalized using a traditional grayscale histogram equalization method and the chromatic channel was processed in a way similar to image warping. The algorithm works in the xy -chromaticity diagram and consists of two steps. In the first step, each color pixel is transformed into its maximally saturated value with respect to a certain color gamut. In the second step, the new color is desaturated toward a new white point.

3. Color histogram equalization methods

3.1. Separate equalization of the three color components—Method I

This is the most simple approach to color histogram equalization. Since many color images have three color bases, the color of each pixel is represented by a 3-dimensional vector \vec{x} and grayscale histogram equalization is performed in each of the three color components separately.

Grayscale histogram equalization attempts to uniformly distribute the pixel gray levels of an image to all the available gray levels L (e.g. $L = 256$, when 8 bits are used to represent each gray level) [1]. Let us consider the image pixel gray level to be an rv x . The histogram of a gray scale image is the probability density function (PDF) of x defined as

$$f_x(x_k) = P\{x = x_k\} = \frac{N(x_k)}{\sum_{m=0}^{L-1} N(x_m)} \quad \forall k = 0, 1, \dots, L-1, \quad (1)$$

where $N(x_k)$ is the number of pixels with gray level value x_k [1]. The CDF $F_x(x_k)$ of the rv x given by

$$y_k = F_x(x_k) = P\{x \leq x_k\} = \sum_{m=0}^k f(x_m) \quad \forall k = 0, 1, \dots, L-1 \quad (2)$$

defines the transformation function for obtaining the gray levels y_k of the equalized image that are uniformly distributed [22].

For color images, the color of each pixel is assumed to be a random vector $\vec{X} = (x_R, x_G, x_B)^T$, where x_R, x_G, x_B are rvs modeling the Red, Green and Blue components,

respectively. Thus, by applying Eq. (2) the equalized histogram of each color component is estimated.

3.2. 3-D Equalization in the RGB space—Method II

The method proposed by Trahanias and Venetsanopoulos [7] is actually a 3-D histogram specification in the RGB color space with the output histogram being uniform. The method is outlined as follows.

- (1) The original 3-D histogram of the color image is computed.
- (2) The joint CDF is computed both for the original random vector \vec{X} and a random vector $\vec{Y}' = (y'_R, y'_G, y'_B)^T$ that is distributed according to a uniform distribution, using Eqs. (3) and (4), respectively,

$$\begin{aligned} y_{R_k G_s B_t} &= F_{x_R x_G x_B}(x_{R_k}, x_{G_s}, x_{B_t}) = P\{x_R \leq x_{R_k}, x_G \leq x_{G_s}, x_B \leq x_{B_t}\} \\ &= \sum_{i=0}^k \sum_{j=0}^s \sum_{m=0}^t f(x_{R_i}, x_{G_j}, x_{B_m}) \quad \forall k, s, t = 0, 1, \dots, L-1, \end{aligned} \quad (3)$$

$$\begin{aligned} y'_{R_{k'} G_{s'} B_{t'}} &= \sum_{i=0}^{k'} \sum_{j=0}^{s'} \sum_{m=0}^{t'} \frac{1}{L^3} \\ &= \frac{(k'+1)(s'+1)(t'+1)}{L^3} \quad \forall k', s', t' = 0, 1, \dots, L-1, \end{aligned} \quad (4)$$

where $f(x_{R_i}, x_{G_j}, x_{B_m})$ is the joint PDF.

- (3) For each color pixel (R_k, G_s, B_t) , the smallest $(R_{k'}, G_{s'}, B_{t'})$ triplet is chosen so that $y'_{R_{k'} G_{s'} B_{t'}} - y_{R_k G_s B_t} \geq 0$. More precisely, the value of $y_{R_k G_s B_t}$ is initially compared to the value $y'_{R_{k'} G_{s'} B_{t'}}$, and in case $y_{R_k G_s B_t}$ is greater (less) than $y'_{R_{k'} G_{s'} B_{t'}}$, the indices R_k, G_s, B_t are repeatedly increased (decreased), one at a time, until the just mentioned inequality is satisfied. The final triplet values form the color of the equalized image.

3.3. Equalization of the intensity component in the HSI space—Method III

The method in Pitas and Kiniklis [5] studies the geometrical representation of the HSI space and formulates three PDFs for applying histogram equalization on the intensity component, the saturation component, and jointly the intensity and saturation components. More visually appealing results are obtained, when the intensity component is only used. Saturation used either separately or jointly with intensity admits large values [5].

Assuming that the color pixel is modeled by the random vector $\vec{X} = (x_H, x_S, x_I)^T$ the PDF for the intensity component is given by [5]

$$f_{x_I}(x_{I_k}) = \begin{cases} 12 \times x_{I_k}^2 & \text{for } 0 \leq x_{I_k} \leq 0.5 \\ 12 \times (1 - x_{I_k})^2 & \text{for } 0.5 \leq x_{I_k} \leq 1. \end{cases} \quad (5)$$

3.4. 2-D Equalization for intensity and saturation in the HSI space — Method IV

The proposed method works in the HSI color space, where each color pixel is modeled by a random vector $\vec{X} = (x_H, x_S, x_I)^T$. However, since hue is the most basic attribute of color and changing it results in unacceptable color artifacts [5,6,20], the method leaves hue unchanged by considering only the 2-D random vector $\vec{Z} = (x_I, x_S)^T$. Thus, a 2-D histogram should be equalized using the joint CDF $F(\vec{Z}) = F(x_I, x_S) = P\{x_I \leq x_I, x_S \leq x_S\}$.

Equalization is performed on intensity and saturation simultaneously by exploiting the following fact [22]: *Given n arbitrary rvs \mathbf{x}_i , the rvs \mathbf{y}_i formed by*

$$\mathbf{y}_1 = F(\mathbf{x}_1), \quad \mathbf{y}_2 = F(\mathbf{x}_2|\mathbf{x}_1), \dots, \mathbf{y}_n = F(\mathbf{x}_n|\mathbf{x}_{n-1}, \dots, \mathbf{x}_1) \quad (6)$$

are independent and each is uniform in the interval. $(0, 1)$.

Following Eq. (6), the equalized pixel values are described by a random vector $\vec{Y} = (y_I, y_S)^T$, where y_I, y_S are both uniform rvs in $(0, 1)$ obtained by the transformations $y_I = F(x_I)$ and $y_S = F(x_S|x_I)$, respectively,

$$y_{I_k} = F(x_{I_k}) = P\{x_I \leq x_{I_k}\} = \sum_{m=0}^k f(x_{I_m}) = \sum_{m=0}^k P(\mathbf{x}_I = x_{I_m}) \quad (7)$$

$$y_{S_t} = F(x_{S_t}|x_{I_k}) = \sum_{m=0}^t f(x_{S_m}|x_{I_k}) = \sum_{m=0}^t \frac{P(\mathbf{x}_I = x_{I_k}, \mathbf{x}_S = x_{S_m})}{P(\mathbf{x}_I = x_{I_k})}. \quad (8)$$

In Eqs. (7) and (8), $k = 0, 1, \dots, L-1$ and $t = 0, 1, \dots, M-1$, where L and M are the number of discrete levels for intensity and saturation, respectively. If 8 bits are used to represent each color component, then $L = M = 256$ [23].

Probability smoothing

The method of histogram equalization defined by Eqs. (7) and (8) suffers from the *sparse data problem* which

is blamed for the presence of unwanted artifacts in the equalized images. It can easily be seen that working in the IS color space, there are L possible different values for intensity in Eq. (7) and $L \cdot M$ possible different pairs of intensity and saturation in Eq. (8), respectively. The observed color component combinations, however, do not exceed the total number of image pixels N . As a result, many color component combinations are actually unseen events whose probabilities are forced to be zero. This problem can be visually verified by the presence of “gaps” in the histogram of the equalized image. That is, there are empty bins between the very full bins. Fig. 1b depicts such a histogram which is of one color component for reasons of simplicity.

In our approach, to alleviate this problem, we used “probability smoothing”, a well-known technique which is largely applied in statistical language modeling for counteracting the effects of statistical variability, that turns up in small data sets [24,25]. Smoothing is based on discounting by which the relative frequencies of seen events are discounted and the gained probability mass is then redistributed over the unseen events. The basic discounting methods for conditional probabilities are Katz’s discounting model, absolute discounting, and linear discounting, each with several variations for the estimation of the discounting parameters. Moreover, probability smoothing methods are based either on backing-off, which amounts to a strict choice between a specific and a generalized probability, or interpolation, where the two probability distributions are added subject to a normalization constraint [25].

The selection of the appropriate smoothing method depends on both the application and the data. In our case, the back-off model of absolute discounting was selected which leaves the high counts virtually unchanged. More precisely, since we have two probability distributions to estimate which are interdependent, a multi-level smoothing was conducted in order to recursively smooth the higher order back-off probability distribution by means of the immediate lower order probability distribution. For $k = 0, 1, \dots, L-1$ and $t = 0, 1, \dots, M-1$ the resulted probabilities are given by:

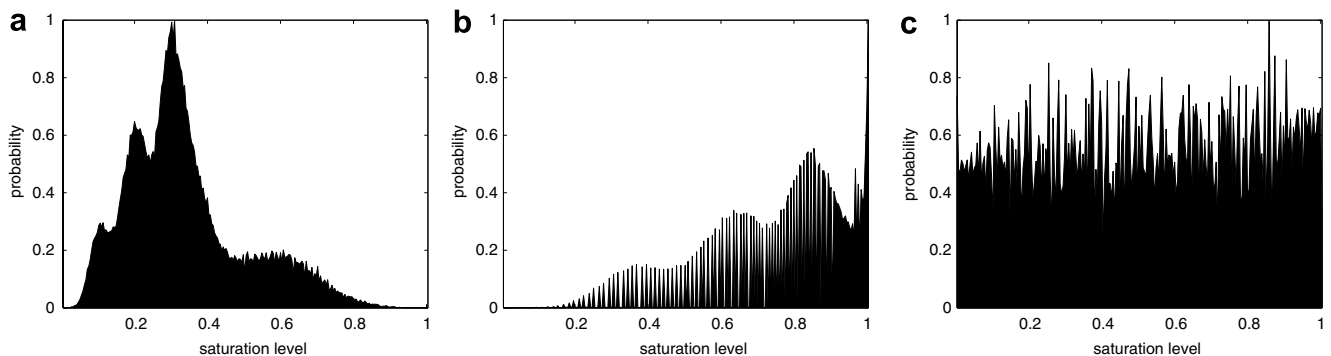


Fig. 1. Histogram of saturation: (a) for the original image, (b) for the equalized image without probability smoothing, and (c) for the equalized image with probability smoothing.

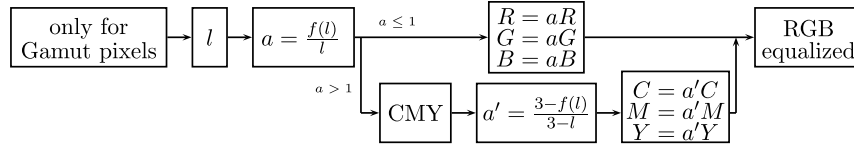


Fig. 2. Block diagram of the proposed equalization method with gamut elimination.

$$P(\mathbf{x}_I = x_{I_k}) = P(x_{I_k}) = \begin{cases} \frac{N(x_{I_k}) - b_I}{N} & \text{if } N(x_{I_k}) > 0 \\ b_I \frac{L - n_0}{N} \frac{1}{\sum_{i: N(x_{I_i})=0} 1} & \text{if } N(x_{I_k}) = 0, \end{cases} \quad (9)$$

$$P(\mathbf{x}_S = x_{S_i} | \mathbf{x}_I = x_{I_k}) = P(x_{S_i} | x_{I_k}) = \begin{cases} \frac{N(x_{I_k}, x_{S_i}) - b_S}{N(x_{I_k})} & \text{if } N(x_{I_k}, x_{S_i}) > 0 \\ b_S \frac{M - n_0(x_{I_k})}{N(x_{I_k})} \frac{P(x_{S_i})}{\sum_{i: N(x_{I_k}, x_{S_i})=0} P(x_{S_i})} & \text{if } N(x_{I_k}, x_{S_i}) = 0. \end{cases} \quad (10)$$

The probability $P(x_{S_i})$ in Eq. (10) is smoothed in a similar way to Eq. (9). In Eqs. (9) and (10), the following notation has been used for the counts and count—counts:

- $L = M = 256$ is the number of discrete levels of intensity and saturation,
- N is the number of image pixels,
- $N(\cdot)$ is the number of pixels with the denoted values for the specified color component(s),
- n_0 is the number of intensity values that are not seen in the image,
- $n_0(x_{I_k})$ is the number of the saturation values that are never seen given that the value of the intensity equals x_{I_k} , $k = 0, 1, \dots, L - 1$,
- $n_r^{(I)} = \sum_{i: N(x_{I_i})=r} 1$ is the number of intensity values that are seen exactly r times,
- $n_r^{(S)} = \sum_{i,j: N(x_{I_i}, x_{S_j})=r} 1$ is the number of (intensity, saturation) pairs that are seen exactly r times,
- $b_I = \frac{n_1^{(I)}}{n_1^{(I)} + 2n_2^{(I)}}$,
- $b_S = \frac{n_1^{(S)}}{n_1^{(S)} + 2n_2^{(S)}}$.

By applying Eqs. (9) and (10) to estimate the joint probabilities that appear in Eqs. (7) and (8), the equalized image histogram is produced. As can be seen in Fig. 1c, the resulting equalized histogram resembles more the uniform pdf and it has all color component values. However, in some cases due to the transformation from the one color space to an other, colors out of the color gamut may be produced yielding equalized images with unwanted color artifacts.

Summarizing, the outline of Method IV is described in the following steps:

- (1) The original RGB image is converted into the HSI color space using the transformation described in Sangwine and Horne [2].
- (2) Histogram equalization is performed on the intensity using Eq. (7) and jointly for the intensity and saturation using Eq. (8). The probabilities that appear in the aforementioned equations are smoothed by Eqs. (9) and (10).
- (3) The equalized image is converted back to the RGB space using the transformation defined in Sangwine and Horne [2].

3.5. 2-D Equalization in Intensity-saturation components of the HSI space with gamut elimination—Method V

The second method proposed in the paper (referred to as Method V) deals with the gamut problem identified previously in Method IV. The method is hue preserving. More specifically, the nonlinear transformation proposed in Naik and Murthy [6] is applied. Let us first briefly describe the application of the nonlinear transformation for contrast enhancement. It is based on the so called S-type transformation defined by

$$f_{m,n}(x) = \begin{cases} \delta_1 + (m - \delta_1) \left(\frac{x - \delta_1}{m - \delta_1} \right)^n, & \delta_1 \leq x \leq m \\ \delta_2 - (\delta_2 - m) \left(\frac{\delta_2 - x}{\delta_2 - m} \right)^n, & m \leq x \leq \delta_2, \end{cases} \quad (11)$$

where x represents the gray scale pixel value and $m \in [\delta_1, \delta_2]$, $n \in (0, \infty)$ are two constants. For the standard S-type contrast enhancement, $n = 2$, $\delta_1 = 0$, $\delta_2 = 3$, and $m = 1.5$. The algorithm defines $\alpha(l_{\bar{x}}) = \frac{f(l_{\bar{x}})}{l_{\bar{x}}}$, where $l_{\bar{x}}$ is the pixel intensity value normalized to 1. That is, $l_{\bar{x}} = r + g + b$, where $r = \frac{x_R}{\max x_R}$, $g = \frac{x_G}{\max x_G}$ and $b = \frac{x_B}{\max x_B}$ are the normalized RGB values in the interval $[0, 1]$. When $\alpha(l_{\bar{x}}) > 1$, which means that the pixel value may be out of gamut, the color vector is transformed to the CMY space (using the equations $x_C = 1 - x_R$, $x_M = 1 - x_G$, and $x_Y = 1 - x_B$) and the new pixel value for each color component is scaled by means of $\alpha(l_{\bar{x}}) = \frac{3 - f(l_{\bar{x}})}{3 - l_{\bar{x}}}$. Then, a transformation back to the RGB space is applied.

This approach is used in combination with the histogram equalization Method IV for the equalized pixel values outside the gamut of RGB, as depicted in Fig. 2.

4. Experimental results

The histogram equalization methods described in Section 3 were implemented and applied to different color

images in order to make a comparative quality assessment study of their performance. The quality of the equalized images was judged both in a subjective way from their visual appeal and the presence of unwanted color artifacts as well as by using objective statistical measures, such as the entropy and the Kullback–Leibler divergence.

The entropy represents the average uncertainty of a random variable and is maximized for the uniform distribution [24,26]. Therefore it consists a good measure since greater entropy values show a more uniform distribution. The entropy of a n discrete rvs is defined by

$$H(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) = - \sum_{x_1} \sum_{x_2} \dots \sum_{x_n} P(x_1, x_2, \dots, x_n) \times \log_2 P(x_1, x_2, \dots, x_n). \quad (12)$$

In a similar way, the Kullback–Leibler divergence measures the difference between two probability distributions [24,26]. In our experiments, the Kullback–Leibler divergence was used in order to measure how similar the histograms of the original and the equalized images are to the uniform distribution. The image probabilities which were taken under consideration were those used for the entropy estimates. That is, for the n -dimensional case

$$D(f(x_1, x_2, \dots, x_n) || g(x_u, y_u, \dots, z_u)) = \sum_{x_1} \sum_{x_2} \dots \sum_{x_n} P(x_1, x_2, \dots, x_n) \times \log_2 \left(\frac{P(x_1, x_2, \dots, x_n)}{g(x_1, x_2, \dots, x_n)} \right), \quad (13)$$

where $g(x_1, x_2, \dots, x_n)$ is a n -dimensional uniform distribution defined in the same space with $f(x_1, x_2, \dots, x_n)$.

Representative experimental results are presented for five color images, namely an indoor scene (Index 1) and a set of four digitalized Orthodox Holy Icons (Indices 2–5). The images are depicted in Figs. 3–7 together with their histograms for the intensity level for the methods that work on the RGB space (Methods I and II) and the intensity and saturation level for the methods that work on the HSI space (Methods III and IV).

An empirical simple comparison of the equalized images (Figs. 3–7) shows that the proposed Method IV produces the most visually appealing results, while the slightest change in the original image and its histogram is observed when the equalization Method II is applied. The success of the proposed method can be attributed to the fact that the method spreads the intensity histogram of the equalized image along the x -axis preserving its shape, unlike Method I and III that distort the original histogram shape. Moreover, the effect of probability smoothing (Method IV) to the elimination of gaps in the histogram for saturation component is obvious. The further improvement of the proposed method on the elimination of the gamut problem using Method V is demonstrated in Fig. 8. Gamut elimination is evident both from images and their 3-D histograms in the RGB space. More precisely, the largely erroneous pixel values in the image which correspond to the outliers

of the 3-D histogram in the Red. Blue hyperplane of Fig. 8b are eliminated in Fig. 8c.

The objective measures of entropy and Kullback–Leibler divergence that are presented, respectively, in Tables 3 and 4, validate the aforementioned empirical results. More precisely, the proposed Method IV compared to all the other methods achieves a higher increase in entropy between the original and the equalized images, which means that the proposed method produces a more uniform histogram. For Method IV the increase ranges from 6 to 12%, while for Method V from 4 to 8%. On the contrary, for Methods I and II there is a slight decrease of entropy between the original and the equalized images. In Method III, the entropy either increases or decreases slightly. The Kullback–Leibler divergence results for the equalized images, which are summarized in Table 4, show that the proposed Methods IV and V compared to the other three methods achieve a higher decrease in Kullback–Leibler divergence (22 to 36% and 16 to 30%, respectively), meaning that the histograms of the equalized images by the proposed methods are more similar to the uniform distribution than the histograms of the equalized images obtained the other methods compared. For Method III, the Kullback–Leibler divergence values either increase or decrease, but this change is negligible.

5. Conclusions

In this paper, two novel color histogram equalization methods are proposed, which work on the intensity and saturation components of the HSI color space. The first method (Method IV) uses probability smoothing to derive the transformations of the original intensity and saturation color components to uniformly distributed ones. The second method (Method V) exploits the empirical technique proposed in Naik and Murthy [6] in order to deal efficiently with the gamut problem that may appear due to the transformation from HSI color space to the RGB space.

The experimental results have demonstrated the superiority of the proposed methods in producing more visually appealing images. The equalized images, as is shown by measuring their entropy and Kullback–Leibler divergence, have more uniform histograms. Moreover, gamut elimination, which is a factor that can be blamed for unwanted color artifacts in images, is also obtained by applying the second method.

Appendix A. Color spaces

The term *color space* or *color model* refers to an abstract mathematical model describing the way a color can be represented as tuple of numbers. The color spaces in use today are oriented either toward hardware, such as color monitors or printers, or toward applications that deal with color manipulation, such as graphics. [23]. Especially, for image processing the RGB and HSI color models are the most frequently used [23].

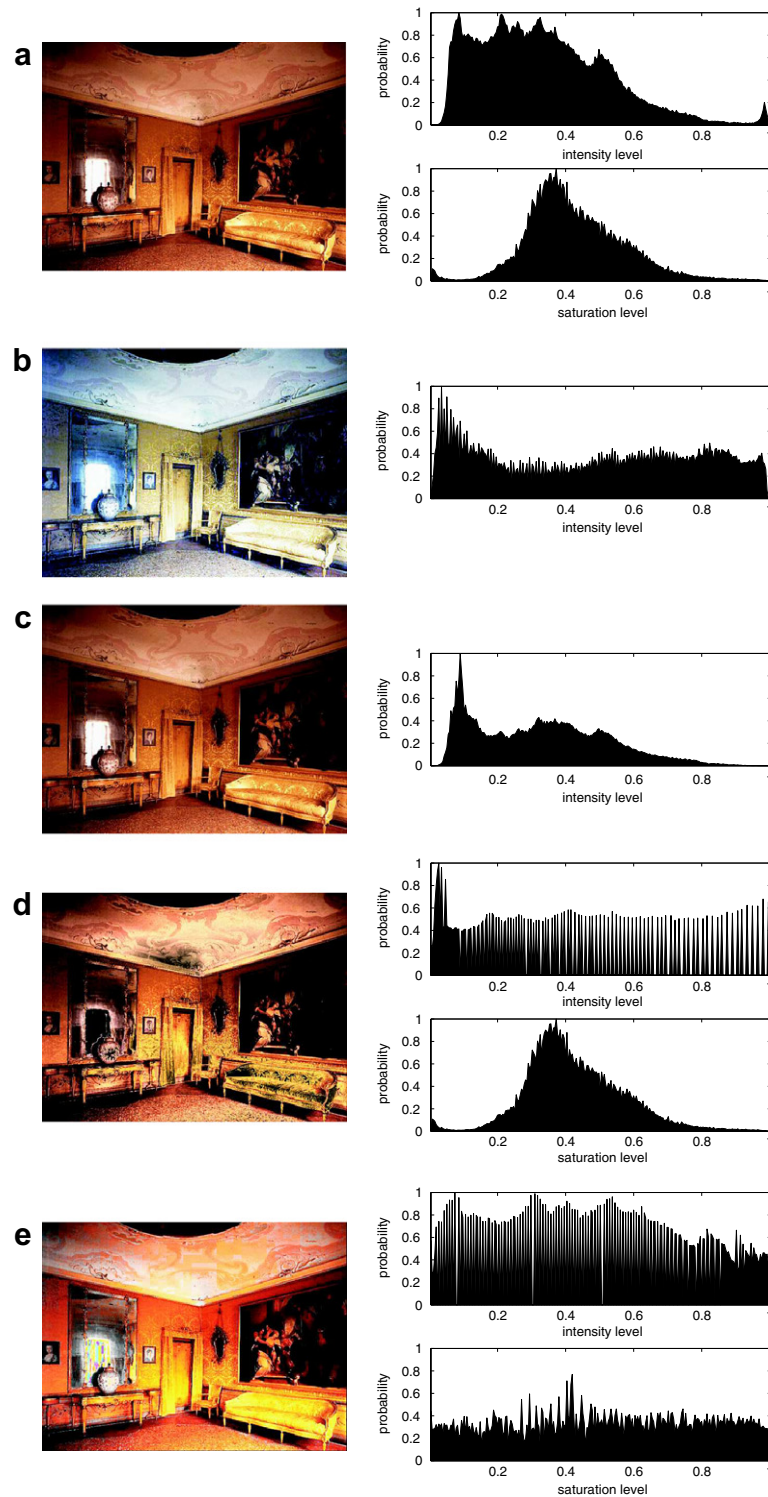


Fig. 3. (a) Original Image (Index 1); (b) Equalized Image by Method I; (c) Equalized Image by Method II; (d) Equalized Image by Method III; (e) Equalized Image by Method IV.

A.1. RGB Color space

In the RGB color space, each color appears in its primary spectral components of Red, Green, and Blue, and it is represented by a point on or inside the unit cube which is depicted in Fig. A.1a [27–29]. It is worth mention-

ing that the values of R, G, and B are normalized to $[0, 1]$ for convenience. Black is at the origin of the cube while all gray colors lie on the main diagonal from black to white [23]. The main disadvantage of the RGB space involving natural images is the high correlation between its components. The RGB space also suffers from non-uniformity,

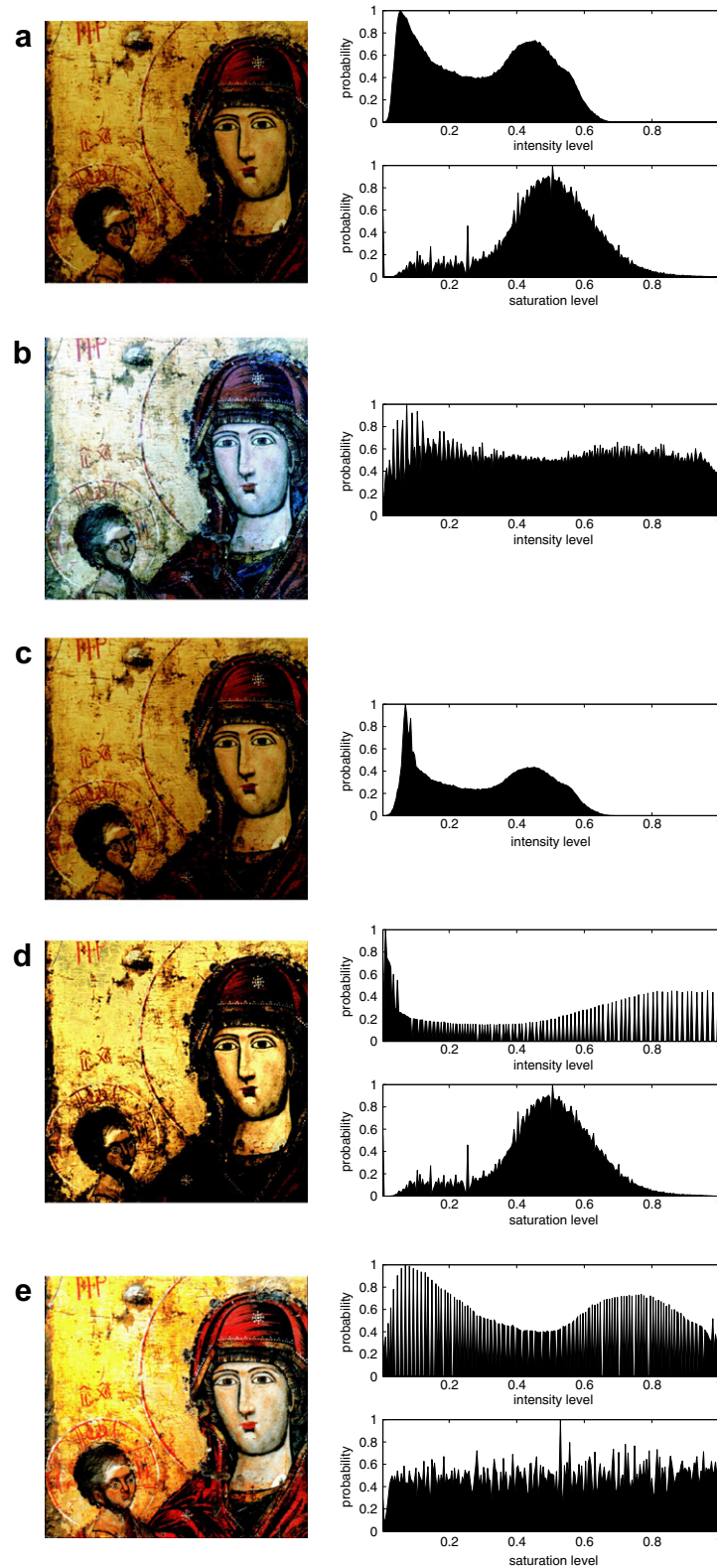


Fig. 4. (a) Original Image (Index 2); (b) Equalized Image by Method I; (c) Equalized Image with Method II; (d) Equalized Image by Method III; (e) Equalized Image by Method IV.

since it is impossible to evaluate the perceived differences between colors on the basis of distances, and psychological

intuitivity, since the visualization of a color based on R, G, B components is rather hard [2].

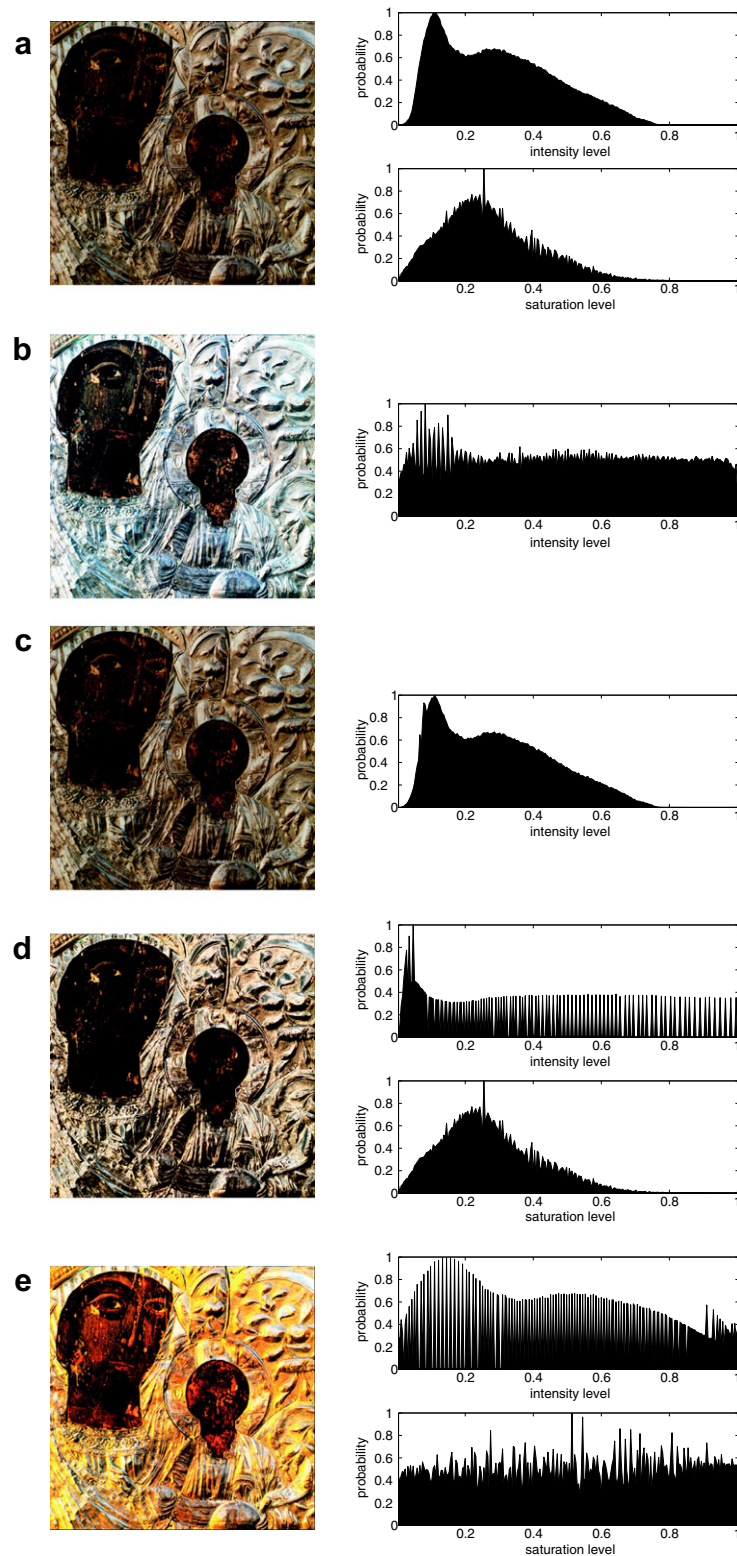


Fig. 5. (a) Original Image (Index 3); (b) Equalized Image by Method I; (c) Equalized Image with Method II; (d) Equalized Image by Method III; (e) Equalized Image by Method IV.

In colorimetry in order to eliminate the influence of illumination intensity incident on the scene represented by an image, chromaticity coordinates were introduced. These

coordinates are actually the normalized values for each color component and they are more stable to changes in illumination level than the original RGB values [2].

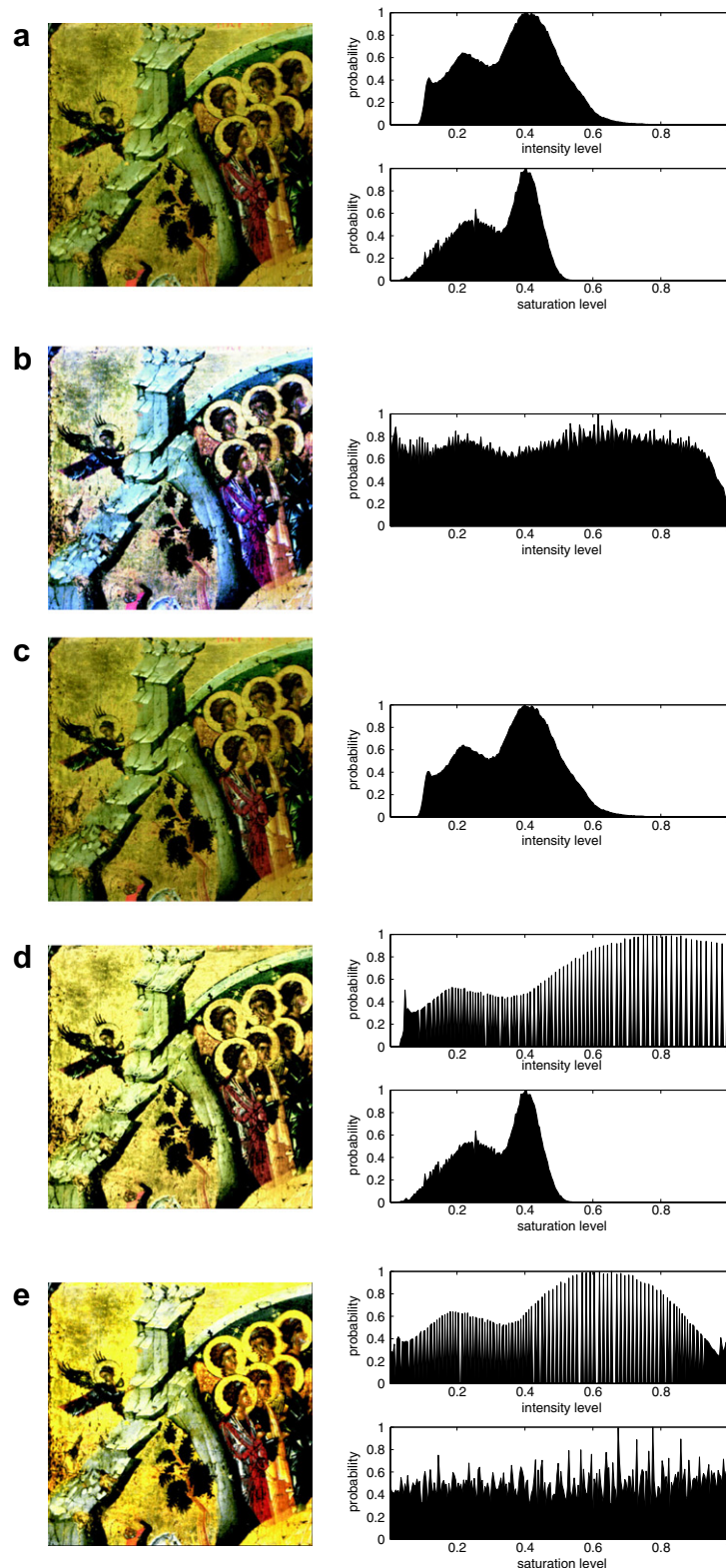


Fig. 6. (a) Original Image (Index 4); (b) Equalized Image by Method I; (c) Equalized Image with Method II; (d) Equalized Image by Method III; (e) Equalized Image by Method IV.

A.2. HSI Color space

The HSI color model is depicted in Fig. A.1b. It is more intuitive to human vision and has many variants, such as

HSB (hue, saturation, brightness), HSL (hue, saturation, lightness), and HSV (hue, saturation, value) [29]. It separates the color information from its intensity information. Intensity is achromatic and describes the brightness of

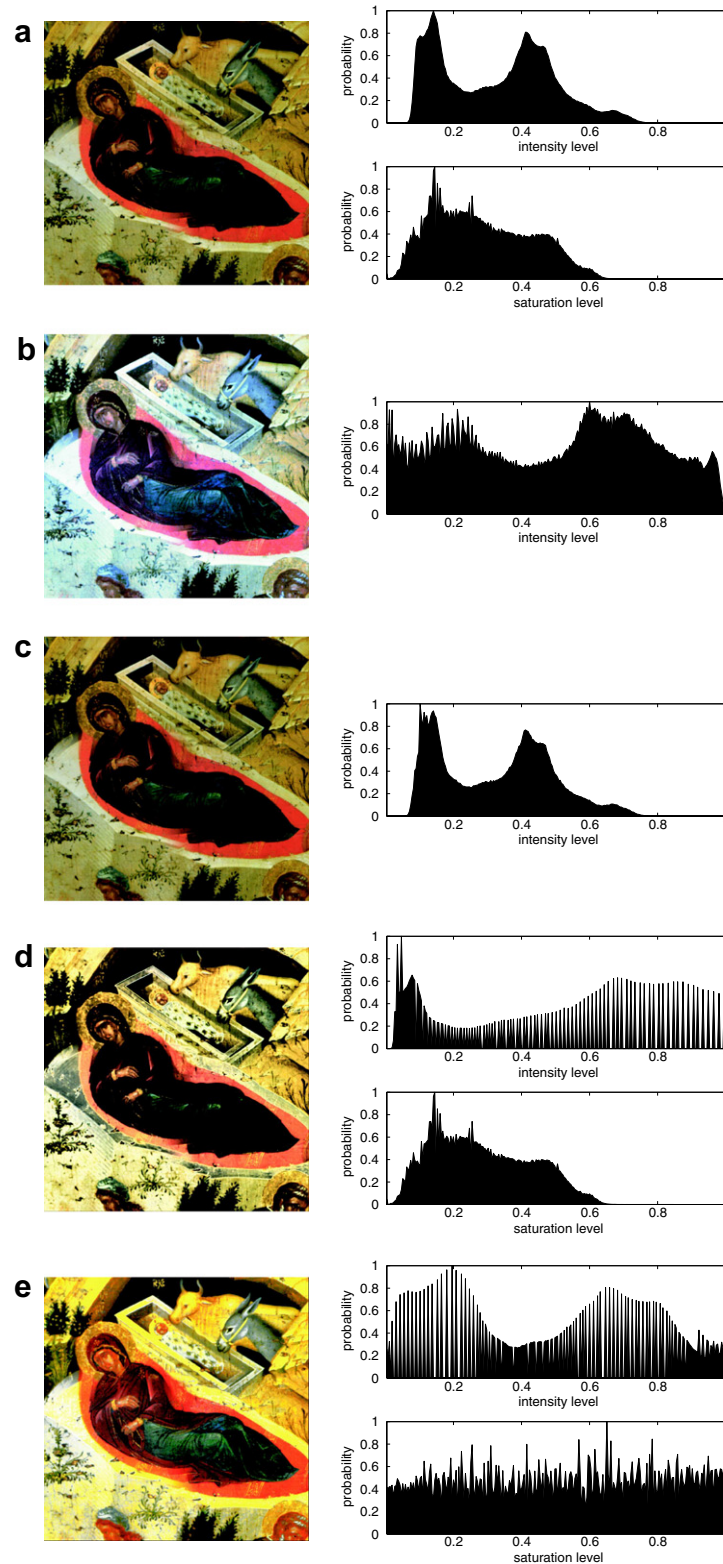


Fig. 7. (a) Original Image (Index 5); (b) Equalized Image by Method I; (c) Equalized Image with Method II; (d) Equalized Image by Method III; (e) Equalized Image by Method IV.

the scene, while hue and saturation are the chromatic components. More precisely, hue is an attribute associated with the dominant wavelength, and thus represents the

dominant color perceived by an observer. Saturation corresponds to relative color purity, that is the amount of white light mixed with a hue [2,23]. That is, in the case of

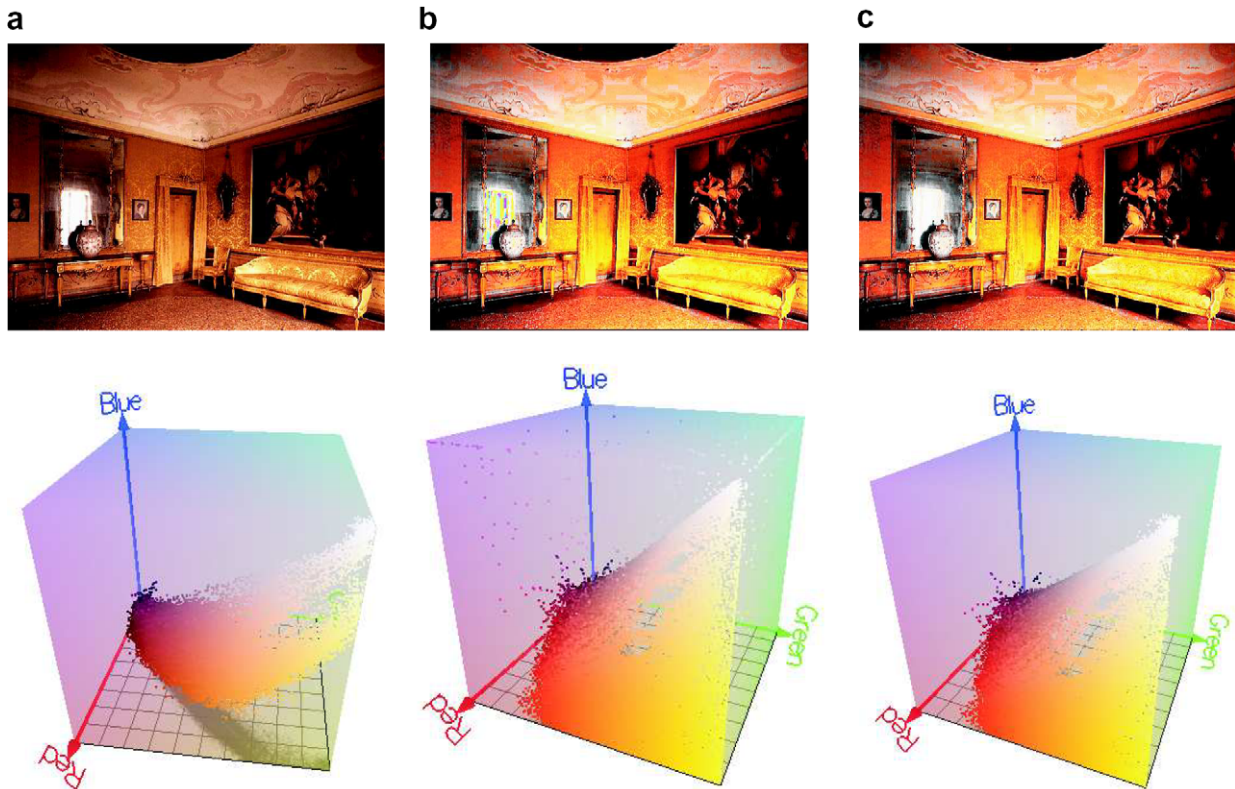


Fig. 8. (a) Original Image (Index 1); (b) Equalized Image by Method IV; (c) Equalized Image by Method V.

Table 3

Entropy of the (a) 3-D color histogram distribution for the original and the equalized images obtained by Methods I and II [7], (b) bivariate color histogram distribution of intensity and saturation for the original and the equalized images obtained by Method III ([5]) and the proposed Methods IV and V

Image index	Original image	Equalized image by method		Image index	Original image	Equalized image by method		
		I	II			III	IV	V
(a)				(b)				
1	14.603	14.588	14.431	1	12.389	12.195	13.215	12.991
2	14.857	14.824	14.797	2	11.952	11.646	13.296	13.132
3	14.489	14.483	14.358	3	11.878	12.048	13.411	13.114
4	15.708	15.708	15.568	4	11.940	12.479	13.416	12.710
5	15.802	15.787	15.622	5	12.002	12.379	13.459	13.041

Table 4

Kullback–Leibler divergence for the (a) 3-D color histogram distribution for the original and the equalized images obtained by Methods I and II [7], (b) bivariate histogram for the original and the equalized images obtained by Method III ([5]) and the proposed Methods IV and V

Image index	Original image	Equalized Image by Method		Image index	Original image	Equalized image by Method		
		I	II			III	IV	V
(a)				(b)				
1	6.514	6.633	6.524	1	2.503	2.637	1.930	2.086
2	6.338	6.379	6.360	2	2.806	3.018	1.874	1.988
3	6.593	6.683	6.597	3	2.858	2.740	1.795	2.001
4	5.748	5.845	5.748	4	2.815	2.440	1.791	2.280
5	5.682	5.807	5.693	5	2.771	2.510	1.761	2.051

a pure color saturation is 100% while colors with zero saturation are gray levels.

Generally, hue is considered as an angle between a reference line and the color point in RGB space. The range of

the hue value is from 0 to 360. The saturation component represents the radial distance from the center. The nearer the point is to the center, the lighter the color is. Intensity is the height in the axis direction, while the axis describes

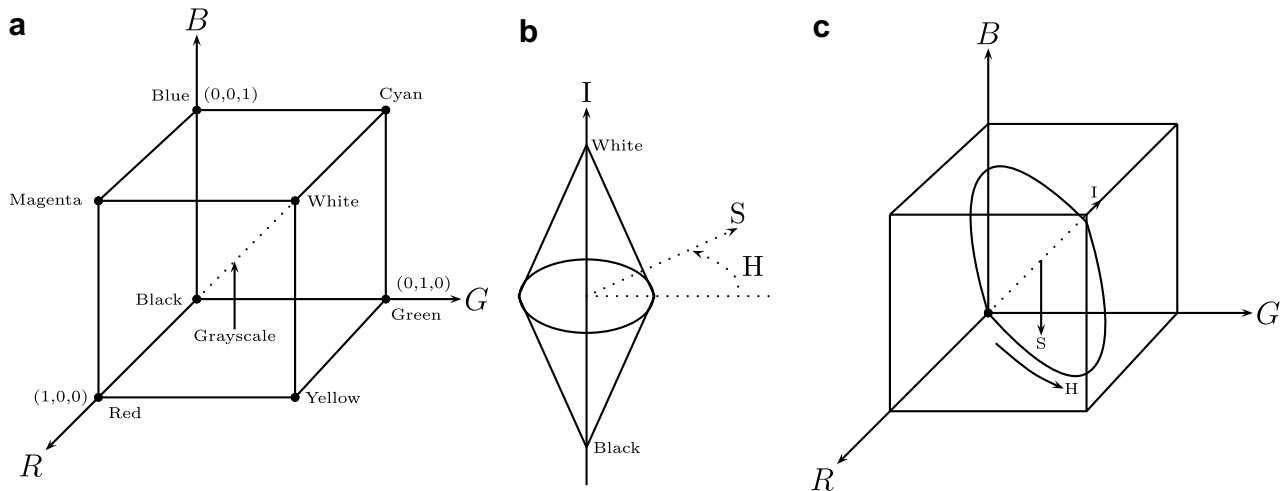


Fig. A.1. (a) The RGB color cube; (b) The HSI color space; (c) RGB-HSI relationship.

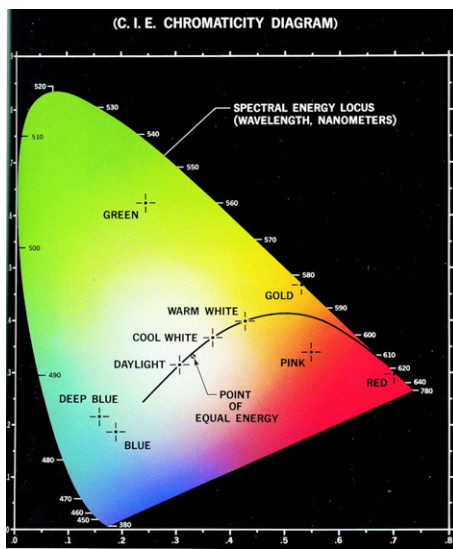


Fig. A.2. Color gamut diagram [23].

the gray levels (i.e. zero for minimum intensity corresponding to black, and full maximum intensity corresponding to white). Each plane perpendicular to the intensity axis is a plane with the same intensity [23,29].

The relationship between the two color spaces is depicted in Fig. A.1c [29]. As it can be easily seen, the intensity is measured along the diagonal of the RGB cube (i.e., the line segment from $(R, G, B) = (0, 0, 0)$ to $(1, 1, 1)$, and hue and saturation are polar coordinates in the plane perpendicular to the I diagonal. The conversion formulae from RGB to HSI and vice versa are rather complex and are found in the literature in different forms [2,23].

The main advantages of the HSI space is its good compatibility with human intuition and the separability of chromatic values from achromatic ones. However, it suffers from the irreducible singularities of the RGB to HSI transformation and the sensitivity to small deviations of RGB values near singularities [23].

A.3. Color gamut

Each color model uses a different color representation. The term *color gamut* is used to denote the universe of colors that can be created or displayed by a given color system or technology. The colors that are perceivable by the human visual system fall within the boundaries of the horse-shoe shape derived from the CIE-XYZ color space diagram, while the RGB colors (that can be displayed on an RGB monitor) fall within the red triangle that connects the RGB primary dots (Fig. A.2). It is obvious that, the full range of perceptible color by humans is not available by the RGB color model and the transformations from one space to another may create colors outside the color gamut [2]. To alleviate this problem various gamut mapping techniques are applied that aim at replacing the out-of-gamut values with substitute attainable values [30].

References

- [1] A.K. Jain, Fundamentals of Digital Image Processing, Prentice-Hall, Englewood Cliffs, N.Y., 1986.
- [2] S.J. Sangwine, R.E.N. Horne, The Colour Image Processing Handbook, Chapman and Hall, 1998.
- [3] S.M. Pizer, E.P. Amburn, J.D. Austin, R. Cromartie, A. Geselowitz, B. Romeny, J.B. Zimmermann, K. Zuiderveld, Adaptive histogram equalization and its variations, Comput. Vis. Graph. Image Process. 39 (1987) 355–368.
- [4] A. Carbonaro, P. Zingaretti, A comprehensive approach to image-contrast enhancement Proc. Int. Conf. Image Analysis Process., IEEE Computer Society, Washington, DC, USA, 1999, p. 241.
- [5] I. Pitas, P. Kiniklis, Multichannel techniques in color image enhancement and modeling, IEEE Trans. Image Process. 5 (1) (1996) 168–171.
- [6] S.K. Naik, C.A. Murthy, Hue-preserving color image enhancement without gamut problem, IEEE Trans. Image Process. 12 (12) (2003) 1591–1598.
- [7] P.E. Trahanias, A.N. Venetsanopoulos, Color image enhancement through 3-d histogram equalization, in: Proc. 11th IAPR Conf. Pattern Recogn., vol. III, 1992, pp. 545–548.

- [8] J.M. Soha, A.A. Schwartz, Multidimensional histogram normalization contrast enhancement, in: *Proc. 5th Canadian Symp. Remote Sensing*, 1978, pp. 86–93.
- [9] R.N. Niblack, *An Introduction to Digital Image Processing*, Prentice-Hall, 1986.
- [10] P.A. Mlsna, J.J. Rodriguez, A multivariate contrast enhancement technique for multispectral images, *IEEE Trans. Geosci. Remote Sensing* 33 (1) (1995) 212–216.
- [11] P.A. Mlsna, Q. Zhang, J.J. Rodriguez, 3-d histogram modification of color images, in: *Proc. IEEE Int. Conf. Image Process.*, vol. III, 1996, pp. 1015–1018.
- [12] Q. Zhang, P.A. Mlsna, J.J. Rodriguez, A recursive technique for 3-d histogram enhancement of color images, in: *Proc. IEEE Southwest Symp. Image Analysis Interpret.*, 1996, pp. 218–223.
- [13] E. Pichon, M. Niethammer, G. Sapiro, Color histogram equalization through mesh deformation, in: *Proc. IEEE Int. Conf. Image Process.*, vol. II, 2003, pp. 117–120.
- [14] F. Pitie, A.C. Kokaram, R. Dahyot, N-dimensional probability density function transfer and its application to color transfer, in: *Proc. IEEE Int. Conf. Comput. Vis.*, vol. II, 2005, pp. 1434–1439.
- [15] T. Kim, H.S. Yang, Colour histogram equalisation via least-squares fitting of isotropic gaussian mixture to uniform distribution, *IEE Electr. Lett.* 42 (8) (2006) 452–454.
- [16] A.K. Forrest, Colour histogram equalisation of multichannel images, *IEE Proc. Vis. Image Signal Process.* 152 (6) (2005) 677–686.
- [17] I.M. Bockstein, Color equalization method and its application to color image processing, *J. Opt. Soc. Am.* 3 (5) (1986) 735–737.
- [18] W.F. McDonnell, R.N. Strickland, C.S. Kim, Digital color image enhancement based on the saturation component, *Opt. Eng.* 26 (7) (1987) 609–616.
- [19] A.R. Weeks, G.E. Hague, H.R. Myler, Histogram specification of 24-bit color images in the color difference (C-Y) color space, *J. Electron. Imag.* 4 (1) (1995) 15–22.
- [20] J. Duan, G. Qiu, Novel histogram processing for colour image enhancement, in: *Proc. IEEE Int. Conf. Image Graph.*, 2004, pp. 55–58.
- [21] L. Lucchese, S.K. Mitra, J. Mukherjee, A new algorithm based on saturation and desaturation in the xy-chromaticity diagram for enhancement and re-rendition of color images, in: *Proc. IEEE Int. Conf. Image Process.*, 2001, pp. 1077–1080.
- [22] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 3rd ed., McGraw Hill, New-York, 1991.
- [23] R.C. Gonzalez, R.E. Woods, *Digital Image Processing*, 2nd ed., Addison Wesley Publishing Company, 1992.
- [24] C.D. Manning, H. Schutze, *Foundations of Statistical Natural Language Processing*, 1st ed., MIT Press, 1999.
- [25] H. Ney, S. Martin, F. Wessel, Statistical language modeling using leaving-one-out, in: S. Young, G. Bloothoof (Eds.), *Corpus-Based Methods in Language and Speech Processing*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997, pp. 174–207.
- [26] R.O. Duda, P.E. Hart, D.G. Stork, *Pattern Classification*, 2nd ed., Wiley Interscience, N.Y., 2000.
- [27] H.R. Kang, *Color Technology for Electronic Imaging Devices*, SPIE Press, 1997.
- [28] K.N. Plataniotis, A.N. Venetsanopoulos, *Color Image Processing and Applications*, 1st ed., Springer, 2000.
- [29] I. Pitas, *Digital Image Processing: Algorithms and Applications*, J. Wiley and Sons, N.Y., 2000.
- [30] E.J. Giorgianni, T.E. Madden, *Digital Color Management: Encoding Solutions*, 1st ed., Addison Wesley Longman, 1998.