Object segmentation and modeling in volumetric images

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Abstract

We propose a pattern classification based approach for simultaneous 3-D object modeling and segmentation in image volumes. The 3-D objects are described as a set of overlapping ellipsoids. The segmentation relies on the geometrical model and graylevel statistics. The extension of the Hough Transform algorithm in the 3-D space by employing the spherical coordinate system is used for ellipsoidal center estimation. The characteristic parameters of the ellipsoids and of the graylevel statistics are embedded in a Radial Basis Function (RBF) network and they are found by means of unsupervised training. We propose a new robust training algorithm for RBF networks based on α-Trimmed Mean statistics. The proposed algorithm is applied for tooth pulpal blood vessel segmentation in a stack of microscopy images.

1 Introduction

Representation and recognition of 3-D objects is an important task in structure identification and visualization [1, 2]. In this study we consider a stack of images, each representing a slice from a 3-D volume. The images composing the stack represent parallel and equidistant cross-sections through the object structure.

We represent the objects with a mixture of overlapping ellipsoids and we consider four input features, denoting the voxel coordinates and the graylevel, for a pattern classification system. The parameters of the ellipsoids can be found using the normalized first and second order moments [3]. In this study we employ RBF's to model the geometry and the graylevel distribution of 3-D objects. In order to estimate the ellipsoids' parameters when it is embedded in noise we employ the α-Trimmed mean algorithm for estimating the RBFs parameters. α-Trimmed Mean algorithm has been shown as efficient in estimating distributions embedded in medium and long tailed noise [4]. The proposed algorithm is analyzed when estimating the parameters of ideal ellipses. The Hough Transform in spherical coordinates is used in the ellipsoids' parameter estimation algorithm. The proposed algorithm contains the classical RBF and Median RBF learning algorithms [5] as particular cases.

2 Segmentation Criterion

A classification approach is employed in the 3-D object segmentation. According to the Bayesian classification theory :

$$p(\mathcal{O}_k|\mathcal{F}) = \max_{j=1}^{M} p(\mathcal{O}_j|\mathcal{F}) \tag{1}$$

where M is the total number of objects, $\mathcal{O}_k$ is the object to be identified and $\mathcal{F}$ is a volumetric image. The *a posteriori* probability is expressed by means of the *a priori* probabilities using the Bayesian rule :

$$p(\mathcal{O}_j|\mathcal{F}) = \frac{p(\mathcal{F}|\mathcal{O}_j)\, p(\mathcal{O}_j)}{p(\mathcal{F})}. \tag{2}$$

We decompose the object probabilities in a set of component probabilities. The relationship (1) becomes :

$$\prod_{m=1}^{L} [p(\mathcal{F}|g_{mk})\, p(g_{mk})] = \max_{j=1}^{M} \left\{ \prod_{n=1}^{L} [p(\mathcal{F}|g_{nj})\, p(g_{nj})] \right\} \tag{3}$$

where the object $\mathcal{O}_j$ is decomposed in the components g_{nj}.

Each of the component probabilities can be expressed as an energy function which is denoted as $\phi_m(\mathbf{X})$, where $\mathbf{X}$ is the feature vector. The *a priori* probabilities $p(g_{nk})$ scaled by a constant are taken as weighting factors denoted by λ_{nj}. The relationship (3) can be expressed as :

$$\sum_{m=1}^{L} \lambda_{mk} \exp[-\phi_m(\mathbf{X})] = \max_{j=1}^{M} \left\{ \sum_{n=1}^{L} \lambda_{nj} \exp[-\phi_n(\mathbf{X})] \right\}, \tag{4}$$

where L is the total number of hidden units. This structure is implemented in an RBF network.

The RBF network consists of a two-layer neural network. A hidden-layer unit implements the Gaussian function :

$$\exp[-\phi_m(\mathbf{X})] = \exp[-(\mathbf{X} - \mu_m)' \mathbf{\Sigma}_m^{-1} (\mathbf{X} - \mu_m)] \tag{5}$$

where μ_m denotes the center vector and $\mathbf{\Sigma}_m$ the covariance matrix. The output of the network is denoted by $Y_k(\mathbf{X})$, where $k = 1, \ldots, N$ and is limited to the interval $(0, 1)$ by a sigmoidal function :

$$Y_k(\mathbf{X}) = \frac{1}{1 + \exp\left(- \sum_{m=1}^{L} \lambda_{mk} \exp[-\phi_m(\mathbf{X})] \right)}. \tag{6}$$

Each output corresponds to an object in the volumetric image.

3 Ellipse parameters estimation

Let us consider the equation of an ellipse in the analytic form :

$$(\mathbf{X} - \mu)'\mathbf{\Sigma}^{-1}(\mathbf{X} - \mu) = 1 \tag{7}$$

where μ denotes the center vector of the ellipse and $\mathbf{\Sigma}^{-1}$ its width and orientation. We denote the components of this matrix as :

$$\mathbf{\Sigma}^{-1} = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \tag{8}$$

We consider an elliptic disc as a uniform distributed statistics inside of an elliptic shape. We can calculate the ellipsis center and width based on the first and second order moments [3].

After trimming a certain percentage from the data associated with an ellipse and applying the first order moment of the resulting distribution we obtain that $E[\hat{\mu}] = \mu$ [3, 6]. This result proves that the estimation of the ellipse center by trimming is unbiased in the case of perfect ellipses. For the covariance matrix we order the data samples of the ellipse according to their distance to the ellipse center. After trimming an α_M percentage from the ordered data we obtain :

$$(\mathbf{X} - \mu)'\mathbf{\Sigma}^{-1}(\mathbf{X} - \mu) = 1 - \alpha_M \tag{9}$$

The area of the ellipse in this case is :

$$\int_{x_{M,inf}}^{x_{M,sup}} \int_{y_{M,inf}}^{y_{M,sup}} dydx = \frac{(1-\alpha_M)\pi}{\sqrt{ab - c^2}} \tag{10}$$

and the numerator from the second order moment :

$$\int_{x_{M,inf}}^{x_{M,sup}} \int_{y_{M,inf}}^{y_{M,sup}} (x - \mu_x)^2 \, dydx =$$

$$\frac{(1-\alpha_M)^2 b}{\sqrt{(ab - c^2)^3}} \int_{-1}^{1} z^2 \sqrt{1 - z^2} dz = \frac{(1-\alpha_M)^2 \pi b}{4\sqrt{(ab - c^2)^3}} \tag{11}$$

where $x_{M,inf}, x_{M,sup}, y_{M,inf}, y_{M,sup}$ are the bounds of the trimmed ellipse.

From (10) and (11) we derive the estimate for the variance :

$$E[\hat{\sigma}_x^2] = \frac{\displaystyle\int_{x_{M,inf}}^{x_{M,sup}} \int_{y_{M,inf}}^{y_{M,sup}} (x - \mu_x)^2 dydx}{\displaystyle\int_{x_{M,inf}}^{x_{M,sup}} \int_{y_{M,inf}}^{y_{M,sup}} dydx} = \frac{(1-\alpha_M)b}{4(ab - c^2)} \tag{12}$$

We get a similar expression for the cross-correlation. In the expression (12), if we consider $\alpha_M = 0$ we obtain the same result as in [3] for estimating the normalized second order moment of an ideal ellipse. In order to correct the ellipse contraction, we use the following expression for estimating the covariance matrix in the α-Trimmed Mean RBF algorithm :

$$\hat{\mathbf{\Sigma}}_j = \frac{\displaystyle\sum_{i=0}^{N_k - 2\alpha_M N_k} (\mathbf{X}_{(i),M} - \hat{\mu}_j)\,(\mathbf{X}_{(i),M} - \hat{\mu}_j)'}{(1-\alpha_M)(N_k - 2\alpha_M N_k)}. \tag{13}$$

4 Modeling 3-D objects using RBF networks

We extend the Hough Transform (HT) [3] in 3-D and use it for identifying the centers of the ellipsoids. In order to find the object edges we apply the 3-D extension of the Sobel edge detector algorithm which provides the edge orientation as well. We consider the spherical coordinate system for the 3-D HT. At the intersections of perpendiculars on the 3-D surfaces, tangent to the significant object edge points, we increment an accumulator. After ordering the data samples $X_{(0)} < \ldots < X_{(N_k)}$ we use the α-trimmed mean algorithm, considering the 3-D HT as well :

$$\hat{\mu}_k(t) = \hat{\mu}_k(t-1) + \frac{c_i\left(\mathbf{u}_{(i)} - \hat{\mu}_k(t-1)\right)}{\sum_{i=\alpha_k N_k}^{N_k - \alpha_k N_k} c_i} \text{ if } \alpha_k N_k < i < N_k - \alpha_k N_k$$
$$\hat{\mu}_k(t) = \hat{\mu}_k(t-1) \text{ otherwise} \quad (14)$$

where the accumulator c_i provided by the Hough Transform is used as a weight and α_k is the data percentage to be trimmed away. By employing the HT, the estimate of the RBF center would coincide with the geometrical ellipsoid center.

The parameter α_k is chosen according to the data distribution. The following measure is used for estimating the tail of the data distribution [7] :

$$Q = \frac{U[0.5] - L[0.5]}{U[0.05] - L[0.05]} \quad (15)$$

where $U[\beta]$, $L[\beta]$ represent the average of the upper and respectively lower β percentage of data samples . The number of data samples to be trimmed away relies directly on the value of Q :

$$\hat{\alpha} = \frac{1-Q}{2}. \quad (16)$$

When the distribution is long tailed, the amount of data samples to be trimmed is large, and when the distribution tail is short, the amount of data samples to be trimmed is small. For the second order statistics parameters, we use :

$$\hat{\alpha}_M = 1 - \frac{U[0.5]}{U[0.05]}. \quad (17)$$

In order to group various ellipsoids in distinct objects we consider a compactness and graylevel similarity criterion. Each voxel in the volumetric image is associated to a certain basis function. Similar regions in the graylevel distribution, are assigned to the same object and they receive the same label. The network output weights λ_{mk} are calculated using backpropagation as in [5].

5 Simulation Results

The proposed algorithm has been tested on a stack of 60 microscopy images, representing the internal structure of a tooth pulp. Some of the frames are represented in Figure 1 (a). A 3-D view of the stack of images is displayed in Figure 1 (b). We intend to segment the blood vessels represented as dark areas. As it can be observed in these images, the tissue structure is very noisy and the objects are not well defined. In the training stage we have used only 20 frames (one out of each three consecutive frames) split in 16×16 blocks. After estimating the parameters from the given subset of data, we apply the model to the entire stack of images. The segmentation results for the frames from Figure 1 (a) when using classical RBF and α-trimmed mean RBF algorithm are shown in Figures 1 (c) and (e). The 3-D segmentation of the two blood vessels is completely automatic and the result of the segmentation provided by α-trimmed mean RBF algorithm is visualized for various perspective angles in Figures 1 (d) and (f). It can be observed from these figures that the 3-D object segmentation is quite accurate.

6 Conclusions

We propose a new algorithm for modeling and segmenting objects in 3-D images. The objects are considered as composed of overlapping ellipsoids. The classifier employed in modeling the 3-D structure and graylevel statistics is the RBF network, where each basis unit corresponds to an ellipsoid. We provide a robust learning algorithm for the RBF network based on the α-Trimmed Mean algorithm. We analyze the performance of the proposed algorithm in estimating parameters of ellipses. In order to find the centers of the ellipsoids we develop a 3-D Hough Transform which is embedded in the training algorithm.

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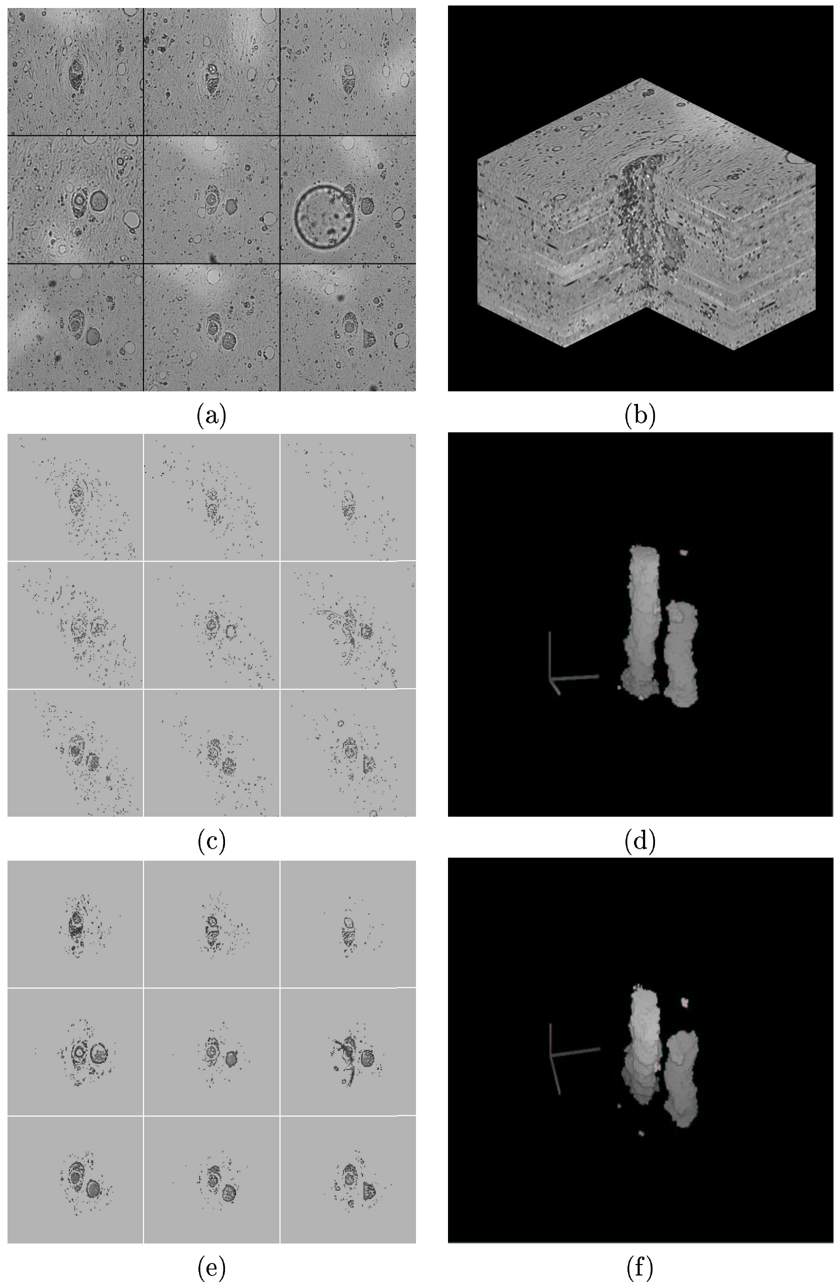

Figure 1: Representations of microscopy images. (a) frames representing tooth pulpal structure ; (b) 3-D visualization of the stack of frames ; (c) segmentation provided by classical RBF algorithm ; (e) segmentation provided by the α-Trimmed Mean RBF algorithm ; (d),(f) blood vessel visualization when the segmentation is provided by α-Trimmed Mean RBF algorithm.