SHAPE - BASED INTERPOLATION USING MORPHOLOGICAL MORPHING

Adrian G. Bors

Dept. of Computer Science
University of York
York YO10 5DD, U.K.

Lefteris Kechagias, Ioannis Pitas

Dept. of Informatics
University of Thessaloniki
Thessaloniki 54633, Greece

ABSTRACT

In this paper we propose an interpolation algorithm using a morphological morphing approach. The aim of this algorithm is to reconstruct an n-dimensional object from a group of $(n-1)$-dimensional sets representing object sections. The morphing transformation modifies consecutive sets so that they approach in shape and size. When the two morphed sets become idempotent we generate a new set. The entire object is modeled by successively interpolating a certain number of intermediary sets between each two consecutive initial sets. The interpolation algorithm is used for 3-D tooth reconstruction.

1. INTRODUCTION

In volumetric images, acquired by magnetic resonance imaging, computer tomography or after mechanical slicing and digitization, the distance between adjacent image elements within a slice can be smaller than the distance between two neighbouring slices. In such situations it is necessary to interpolate additional slices in order to obtain an accurate description of the object for volume visualization and processing. There are two main categories of interpolation techniques for reconstructing objects from sparse sets: grey-level and shape-based interpolation.

Shape-based interpolation algorithms usually consider certain features describing the object shape. The algorithms proposed in [1] employ distance transforms for interpolating new sets by adding or removing layers of elementary units. Other extensions of this algorithm are proposed in [2, 3]. In [4] a morphing algorithm based on a distance transform, namely the skeleton by influence zones (SKIZ), was used for set and function interpolation. We propose a new approach which morphs two consecutive sets one into another. The interpolated set is obtained for the idempotency of the two morphed sets. The interpolation is repeated until we obtain a contiguous volume. We describe the morphological morphing procedure in Section 2 and we show how to interpolate the entire volume in Section 3. Simulation results are provided in Section 4 and the conclusions are drawn in Section 5.

2. MORPHOLOGICAL MORPHING

We employ mathematical morphology operations [5] such as erosions and dilations for morphing consecutive 2-D sets one into another. Let us consider that we are provided with two aligned sets representing two shapes, denoted by X_i and X_{i+1}, partially overlapping, *i.e.* $X_i \bigcap X_{i+1} \neq \emptyset$, in an n-dimensional space denoted as E. Shape morphing is a technique for constructing a sequence of sets representing a gradual transition between the two given shapes. Let us consider $X_{i,m}$ an element (pixel in 2-D or voxel in 3-D) contained into the set X_i, where m denotes an ordering number and $X_i^c = E - X_i$ denotes the complement (background) of the set X_i. Each element $X_{i,m}$ in one set will have a corresponding element, that has the same coordinates, which may be a member of the other set $X_{i+1,m} \in X_{i+1}$, or may be part of its background $X_{i+1,m}^c \in X_{i+1}^c$.

Our morphing transformation ensures a smooth transition from one shape set to the other by means of several sets whose shapes change gradually. Let us denote the boundary set of X_i by C_i. We can identify three possible correspondence cases for the elements of the two aligned sets. One situation occurs when the border region of one set corresponds to the interior of the other set. In this case we dilate the border elements :

$$\begin{array}{ll} \text{If} & X_{i,m} \in C_i \wedge X_{i+1,m} \notin C_{i+1} \\ \text{then} & \text{perform } X_{i,m} \oplus B_1 \end{array} \quad (1)$$

where $\oplus$ represents dilation and B_1 is the structuring element for the set X_i. A second case occurs when the border region of one set corresponds to the background of the other set. In this situation we erode the boundary elements :

$$\begin{array}{ll} \text{If} & X_{i,m} \in C_i \wedge \exists\, X_{i+1,m}^c \\ \text{then} & \text{perform } X_{i,m} \ominus B_1 \end{array} \quad (2)$$

where $\ominus$ denotes erosion. No modifications are performed when both corresponding elements are members of their sets boundary :

$$\begin{array}{ll} \text{If} & X_{i,m} \in C_i \wedge X_{i+1,m} \in C_{i+1} \\ \text{then} & \text{perform no change} \end{array} \quad (3)$$

The last situation corresponds to regions where the two sets coincide locally and no change is necessary, while (1) and (2) correspond to local morphing transformations.

By including all these local changes we define the following morphing transformation of the set X_i depending onto the set X_{i+1} and on the structuring element B_1 :

$$f(X_i|X_{i+1}, B_1) = [(X_i \ominus B_1) \bigcup ((X_i \bigcap X_{i+1}) \oplus B_1)] \bigcap (X_i \bigcup X_{i+1}) \quad (4)$$

A similar morphing operation $f(X_{i+1}|X_i, B_2)$ is defined onto the set X_{i+1} depending on the set X_i. According to these transformations, the intersection $X_i \cap X_{i+1}$ is always retained by the morphing operations while the resulting set is always a subset of $X_i \bigcup X_{i+1}$. The first set will be eroded in those regions which correspond to the background of the second set and will dilate in regions which correspond to the interior of the second set.

The morphing operation applied on either set produces a new set. These morphed sets are closer to each other in shape structure and in size. In order to assess their similarity we define a shape distance. Let us consider a structuring element $B(R)$ as a ball of radius R. This can be obtained from an elementary ball (ball of unit radius) after R successive dilations using the elementary ball as the structuring element. Let us define a shape distance between the original set and the morphed set as given by the size of the structuring element, R. Since each set is ordered according to an index, we can conventionally assume a positive and a negative direction. After morphing the sets X_i and X_{i+1} with the same structuring element $B(R)$, the distance of the morphed sets to their original sets is given by :

$$\begin{aligned} d(f(X_i|X_{i+1}, B(R)), X_i) &= \\ -d(f(X_{i+1}|X_i, B(R)), X_{i+1}) &= R \end{aligned} \quad (5)$$

where the negative distance has been conventionally assigned according to the indexing of sets in the volumetric representation. If we consider identical structuring elements for both sets, each resulting morphed set is equi-distant to its original set according to (5).

3. GEOMETRICALLY CONSTRAINED INTERPOLATION

The morphing operation defined by (4) is applied iteratively onto the sets resulting from previous morphings. The succession of morphing operations creates new sets. With each iteration, these sets are closer in shape and size to each other. The morphing interpolation is based on the following theorem :

Theorem 1 *We can generate an intermediary set between two object sets X_i and X_{i+1}, satisfying $X_i \bigcap X_{i+1} \neq \emptyset$, by iterating the set transformation defined in (4) onto their previous iteration output sets, until idempotency.*

Proof: In order to prove the morphing interpolation convergence to idempotency let us consider a set Y, representing the XOR operation for the two given sets :

$$Y(X_{i+1}, X_i) = (X_{i+1} \bigcup X_i) - (X_{i+1} \bigcap X_i) \quad (6)$$

Let us consider that the local morphing termination condition (3) does not occur at the next morphing iteration, which implies that :

$$[(X_{i+1} \bigcup X_i) \ominus B] \supset [(X_{i+1} \bigcap X_i) \oplus B] \quad (7)$$

In this case, we observe that, by considering $f(X_i|X_{i+1}, B_1)$ from (4) and $f(X_{i+1}|X_i, B_1)$, and by grouping the resulting set components we obtain :

$$\begin{aligned} &Y(f(X_i|X_{i+1}), f(X_{i+1}|X_i)) = [f(X_i|X_{i+1}) \bigcup \\ &f(X_{i+1}|X_i)] - [f(X_i|X_{i+1}) \bigcap f(X_{i+1}|X_i)] \\ &= [((X_{i+1} \bigcup X_i) \ominus B) - ((X_{i+1} \bigcap X_i) \oplus B)] \\ &\bigcap (X_{i+1} \bigcup X_i) = Y(X_{i+1}, X_i) \ominus B. \end{aligned} \quad (8)$$

For the sake of notation simplicity we dropped out the dependency on the elementary structuring element from the expression of the morphing transformation (8). The successive morphing operations preserve the morphing rules outlined in (1), (2) and (3). We can observe that erosion applies everywhere on the set Y, except for the points which fulfill the condition (3). There is a clear interdependency between the set Y defined in (6) and the morphological shape distance defined in (5). While the set Y, with each iteration, is eroded, as it is shown by equation (8), the distance between the resulting sets, morphed from X_i and X_{i+1}, decreases correspondingly :

$$d(f(X_i|X_{i+1}), f(X_{i+1}|X_i)) = d(X_i, X_{i+1}) - 2. \quad (9)$$

Let us denote the morphing at iteration t, initiated from the sets X_i and X_{i+1}, by $f_t(X_i|X_{i+1})$ and $f_t(X_{i+1}|X_i)$, respectively. According to the relationship (9), at each iteration the distance between the morphed sets decreases. The equation (3) represents a local stopping condition which is likely to extend with each iteration to a larger amount of elements from the boundary of the morphed sets. The termination condition of the interpolation algorithm corresponds to the case when the morphing termination condition of (3) is fulfilled for all the boundary points of the two morphed sets. In this situation the two morphed sets become idempotent. Let us consider that this happens after t_1 iterations. Idempotency after t_1 iterations is shown by a zero distance between the resulting morphed sets :

$$d(f_{t_1}(X_i|X_{i+1}), f_{t_1}(X_{i+1}|X_i)) = 0 \quad (10)$$

Let us denote by $\hat{X}_{i+0.5}$ the set resulting at the idempotency of the morphing transformation :

$$\hat{X}_{i+0.5} = f_{t_1}(X_i|X_{i+1}) = f_{t_1}(X_{i+1}|X_i) \quad (11)$$

The resulting set has similarities to both initial sets X_i and X_{i+1}. This set is equidistant to the original sets :

$$d(X_i, \hat{X}_{i+0.5}) = -d(X_{i+1}, \hat{X}_{i+0.5}) \tag{12}$$

The existence of a set which is equidistant to the initial sets and which corresponds to the case when the set $Y(t_1)$ becomes a contour proves the convergence of the morphing *Theorem* 1.

Let us consider that a 3-D object is represented by the group of its cross-section sets. The morphing procedure presented above interpolates a new group of sets between each two consecutive sets. In the general case, each new set is equi-distant to the original neighbouring sets. The initial and the interpolated sets forms a new group of sets which can be used for a better visualization of the given 3-D object. We repeat the same procedure on the new pairs of consecutive sets for modeling the entire object to a finer detail. After K repetitions, the number of interpolated sets generated between two initial sets is $2^K - 1$. Evidently, there is an upper limit in the number of distinctly interpolated sets generated between two given consecutive sets. For N initial sets we obtain $(N-1)(2^K-1)$ interpolated sets. The number of sets to be inserted depends on the relationship between the slice spacing and set element size. In the case of unequally spaced cross-section sets, a different number of sets must be interpolated between each two consecutive slices. Another way to deal with unequally spaced interpolation would be to generate all the possible intermediary sets and to choose certain sets, according to their desired intra-set distance.

4. SIMULATION RESULTS

We have used the proposed morphological morphing interpolation algorithm for reconstructing the external and internal 3-D morphology of several teeth. Such an application is of great interest in endodontology [6]. Two examples are presented in this paper : an incisor (single root tooth) and a molar (three-root tooth). These teeth have been mechanically sliced, digitized and aligned using a semi-automatic procedure. Aligned slices are displayed in Figure 1 for the incisor. We have used the morphological interpolation algorithm described in Sections 2 and 3 for reconstructing the incisor from the given set of slices. In the case of the incisor, the morphing algorithm is applied iteratively four times. Thus we eventually produce $21*(2^4-1)+22 = 337$ slices from only 22 original slices. A set of interpolated frames from the incisor sequence is displayed in Figure 2. We can observe from this figure that both canal and outer tooth surface are being smoothly changed from one slice to the next one. 3-D reconstructions from two different viewing angles are shown in Figures 3a and 3b, for the incisor. A set of slices from a molar are displayed in Figure 4. Two 3-D views of the reconstructed 3-D molar are shown in Figures 5a and 5b. The interpolation of the molar image sequence shows the capability of morphing between slices with disconnected sets and those having compact sets. In both examples the morphology of the reconstructed teeth is quite accurate.

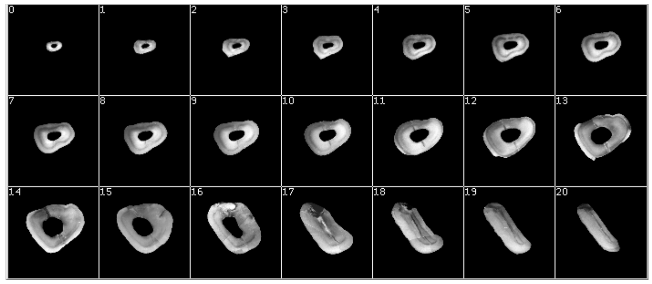


Fig. 1. Segmented and aligned slice sets of an incisor.

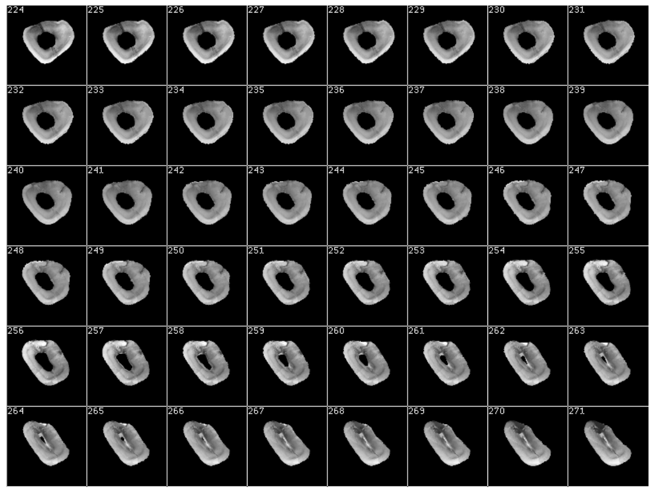
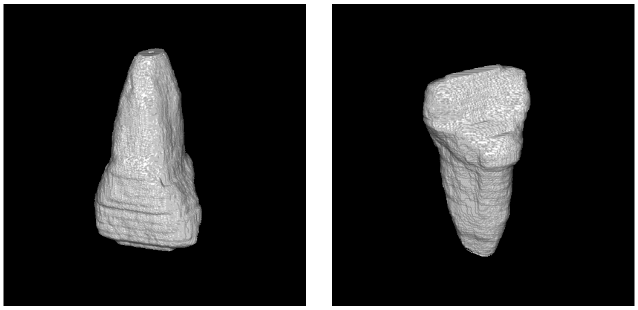


Fig. 2. Interpolated slices.

We compare the mathematical morphological interpolation algorithm with a linear interpolation algorithm. We apply the linear interpolation algorithm on the incisor sequence displayed in Figure 1. In order to compare the two interpolation algorithms we derive an error measure. We consider as a performance measure the percentage of wrongly estimated pixels when comparing X_{i+1} with the set $\hat{X}_{i+1}$ resulted from the interpolation of X_i and X_{i+2} :

$$E = \frac{|Y(\hat{X}_{i+1}, X_{i+1})|}{|X_{i+1}|} \tag{13}$$

where $|X|$ denotes the set cardinality. In Table 1 we provide the results for reconstructing three different slices from the incisor group of sets as well as the average result for reconstructing any intermediary slice X_{i+1} from the given group of sets X_i and X_{i+2} for any $i \in \{1, N-2\}$, where N is the number of initial sets. For comparison we provide the



(a) (b)

Fig. 3. 3-D views of the recovered incisor.

normalized slice difference between the two slices used for interpolation. These results show that the proposed morphological morphing interpolation algorithm provides good experimental results in the case of 3-D tooth reconstruction from slices.

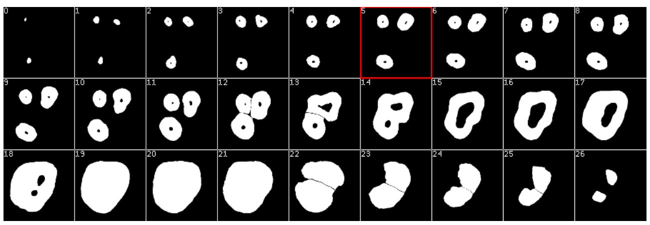


Fig. 4. Initial slices of a molar.

5. CONCLUSIONS

This paper introduces a new interpolation algorithm for reconstructing n-dimensional shapes from $(n-1)$-dimensional sparse sets. The proposed algorithm inserts a new set between each two existing ones by employing a morphological morphing transformation. The new set is equi-distant to the original sets according to a distance measure. The procedure is repeated on the resulting group of sets. The proposed algorithm is applied for 3D teeth reconstruction. The simulation results show good interpolation and robustness even when we have significant variations in shape structure from one set to the next one. The motivation for developing this algorithm is to create a database of various types of teeth. Such tooth volumes can be used for a virtual tooth drilling simulator in pre-clinical dentistry training.

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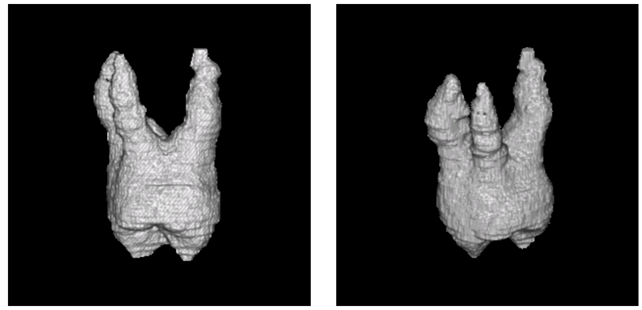

(a) (b)

Fig. 5. 3-D views of the interpolated molar.

Table 1. Comparison between morphological morphing and linear interpolation when reconstructing an incisor.

Frames i,i+1,i+2	Slice Differ. (%) $\frac{\lvert Y(X_{i+2},X_i)\rvert}{\lvert X_i\rvert}$	Morphological Morphing $E(\%)$	Linear Interpolation $E(\%)$
4,5,6	62.9	5.9	11.925
10,11,12	26.8	6.84	9.46
18,19,20	27.2	7.5	14.28
Average	51.5	9.25	11.46

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