PERSPECTIVE DISTORTION ANALYSIS FOR MOSAICING IMAGES PAINTED ON CYLINDRICAL SURFACES

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ABSTRACT

A set of monocular images of a curved painting is taken from different viewpoints around its curved surface. After deriving the surface localization in the camera coordinate system we backproject the image on the curved surface and we flatten it. We analyze the perspective distortions of the scene in the case when it is mapped on a cylindrical surface. Based on the result of this analysis we derive the necessary number of views in order to represent the entire scene depicted on a cylindrical surface. We employ a matching-based mosaicing method for reconstructing the scene from the curved surface. The proposed method is appropriate to be used for painting reconstruction.

1. INTRODUCTION

For each image taken from a different viewpoint we derive its localization. After detecting the projections of two parallels from the curved surface on the image, we calculate their common normal and we derive the localization parameters [1]. By using the localization parameters we backproject the image on the curved surface and afterwards we can flatten the painting representation.

The difference in perspective distortion has been used for computing the shape of the curved surface from texture information [2]. We analyze the geometrical distortions caused by the perspective view in the case of cylindrical surfaces. These distortions are caused by the fact that arcs having different length, project on the image plane in segments of identical size, representing pixels. The distortions are larger in the regions of contact between the tangent from the viewpoint and the curved surface. Based on the perspective distortion analysis we derive the size of the regions which contain big distortions. The reconstruction of the image from the cylindrical surface is done through mosaicing [3] of flattened representations. We employ an automatic mosaicing method based on region matching. The image regions with large distortions, caused by perspective projection, are excluded from matching [4]. We evaluate the bounds of the necessary number of views in order to represent the entire painting on a cylinder. The proposed method is applied in painting visualization and can be further used for painting restoration.

2. CURVED SURFACE LOCALIZATION AND FLATTENING

First, we describe how we can find the localization of the curved surface from a single perspective view. In order to perform the localization, we limit our study to the case when the surface has a curvature different than zero in only one direction. The localization is described by three rotation angles θ_x, θ_y and θ_z. These rotation angles give us the relationship between the camera coordinate system and the coordinate system of the curved surface. Two projection axes must be localized [1, 5]. First we find the projection of the revolution axis, and afterwards we derive the position of the second axis corresponding to the projection of one particular parallel.

In order to find these two axes in the image, we use a certain amount of *a priori* knowledge. We detect two curves in the projected image which are projections of parallels located on the curved surface. After the detection of the two curves we find the projection of the revolution axis in the image. In order to do this, we identify the common normal P_1P_2 of two parallel curves which are projected in the image, as shown in Figure 1 and described in [1]. The slope of the straight line P_1P_2 gives us the direction of the axis. In the image coordinate system (u, v), the equation of this axis is :

$$v = A_1.u + B_1, \qquad (1)$$

where A_1 and B_1 are the coefficients of the straight line P_1P_2. From the equation (1) we derive the rotation angle θ_z corresponding to the angle between u and the revolution axis, as shown in Figure 1 :

$$\theta_z = \arctan(A_1). \qquad (2)$$

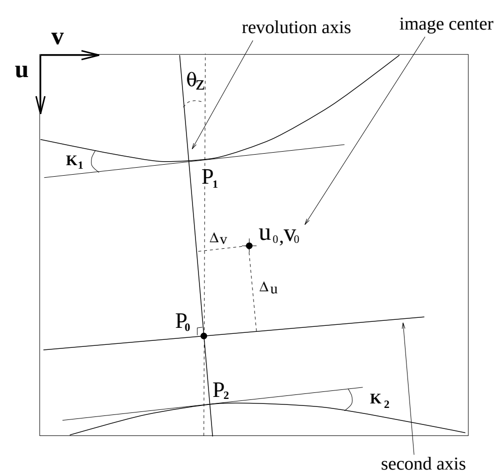

Figure 1. The two axes derived from parallel curves.

Among all the parallels on the curved surface, only one is projected on the image as a straight line. This straight line belongs to the plane passing through that parallel and through viewpoint, and defines the second axis. In order to obtain the two other rotation angles, θ_x and θ_y, let us consider P, a point located on the curve passing through either P_1 or P_2. We define the curvature at a point P_i as :

$$K_i = \lim_{P \to P_i} \frac{\alpha(P) - \alpha(P_i)}{|\widehat{PP_i}|}, \quad (3)$$

where $i \in \{1, 2\}$, $\alpha(P_i)$ is the angle of the tangent to P_i, and $|\widehat{PP_i}|$ is the length of the arc between P and P_i.

We denote by P_0, the point belonging to the revolution axis, where the curvature K_0 equals zero. By considering this in (1) we obtain the equation of the second axis :

$$v = -\frac{1}{A_1}.u + \left(v_0 + \frac{u_0}{A_1}\right), \quad (4)$$

where (u_0, v_0) are the coordinates of the image center. We denote by (Δ_u, Δ_v) the vector distance between (u_0, v_0) and P_0, the intersection of the two axes as shown in Figure 1. The two rotation angles are then given by :

$$\begin{cases} \theta_x = \arctan(\frac{\Delta_v}{f.k}) \\ \theta_y = \arctan(\frac{\Delta_u}{f.k}), \end{cases} \quad (5)$$

where f is the focal distance and k is the resolution factor. Based on the localization parameters we backproject the image on the 3D surface and we match the point P_0 with the image center (u_0, v_0). Then, we obtain the virtual image. In the virtual image, P_0 is projected to the image center and the projection of the revolution axis is vertical. After backprojecting the image onto the curved surface, we flatten the surface in order to obtain a new image without geometrical distortions caused by the surface curvature [4].

3. PERSPECTIVE DISTORTION ANALYSIS

Let us consider the image of a cylindrical surface constructed such that the axes of the camera coordinate system coincide with the object axes. The focal axis z is perpendicular on the revolution axis of the cylinder. The radius of the cylinder is denoted by R, and the viewpoint O is situated at a distance l from the revolution axis of the cylinder, as shown in Figure 2. The projection of the arc $|\widehat{A_1A_{2m}}|$ to the virtual image plane is the line segment $|U_1U_{2m}|$.

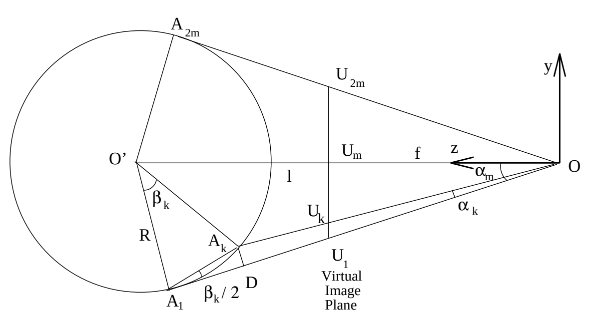

Figure 2. The cross-section representation through cylinder and image plane.

The horizontal cross-section through image is discretized in $2m$ equal-sized intervals :

$$|U_kU_{k-1}| = |U_{k-1}U_{k-2}| = 1 \text{ pixel for } k = 3, \ldots, 2m. \quad (6)$$

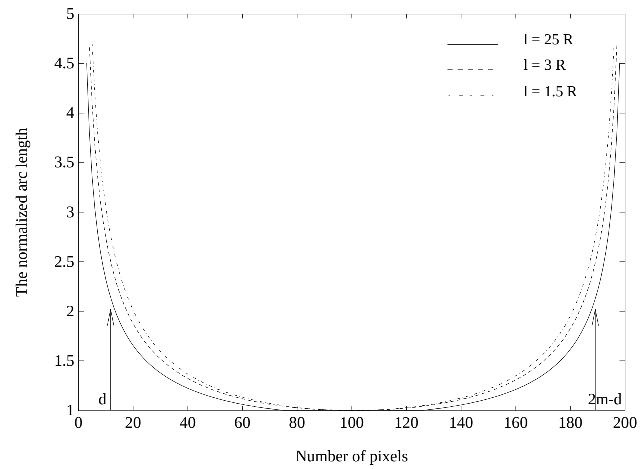

Figure 3. The length of the arcs, each of them corresponding to an equal-sized segment in the image.

Let us denote the angles $\widehat{A_1OA_k}$, $\widehat{A_1O'A_k}$ by α_k and β_k. Based on geometrical considerations we express the length of the line segment $|A_kD|$ in Figure 2, in two different ways. From the triangles $O'A_1A_k$ and A_1A_kD we derive :

$$|A_kD| = 2R\sin^2\left(\frac{\beta_k}{2}\right). \quad (7)$$

$|A_kD|$ is calculated from the triangle A_kDO :

$$|A_kD| = (\sqrt{l^2 - R^2} - R\sin\beta_k)\tan\alpha_k. \quad (8)$$

From the Thales theorem in the triangles OU_1U_m and $OO'A_1$ and by using (6) we find :

$$\tan(\alpha_m - \alpha_k) = \frac{(m-k)}{m\sqrt{\mu^2 - 1}}, \quad (9)$$

where $\mu = l/R$ and α_m denotes the angle $\widehat{O'OA_1}$. From (6) we find $\tan\alpha_k$ with respect to the number of pixels k :

$$\tan\alpha_k = \frac{k\sqrt{\mu^2 - 1}}{m\mu^2 - k}. \quad (10)$$

From (7), (8), and (10), we obtain :

$$2\sin^2\frac{\beta_k}{2} = (\sqrt{\mu^2 - 1} - \sin\beta_k)\frac{k\sqrt{\mu^2 - 1}}{m\mu^2 - k}, \quad (11)$$

for $k = 1, \ldots, 2m$. After the evaluation of the angles β_k from (11), we derive the arc of the cylinder corresponding to an image segment of constant size :

$$|\widehat{A_kA_{k-1}}| = (\beta_k - \beta_{k-1})R. \quad (12)$$

The normalized arc length $|\widehat{A_kA_{k-1}}|$ / $|\widehat{A_mA_{m-1}}|$, calculated from (12) is represented in Figure 3 for $m = 100$ when $\mu \in \{1.5, 3, 25\}$. From this plot we observe that arcs of different lengths from the cylindrical surface are projected to segments with the same length in the image plane. This produces a nonlinear distortion which is larger in the regions situated at the contact of the tangents from the viewpoint to the cylindrical surface (limb points), where $\beta_k \to 0$.

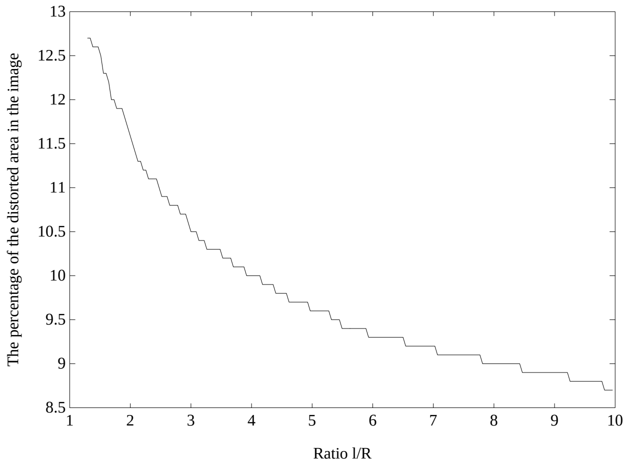

Figure 4. The evaluation of the distorted area.

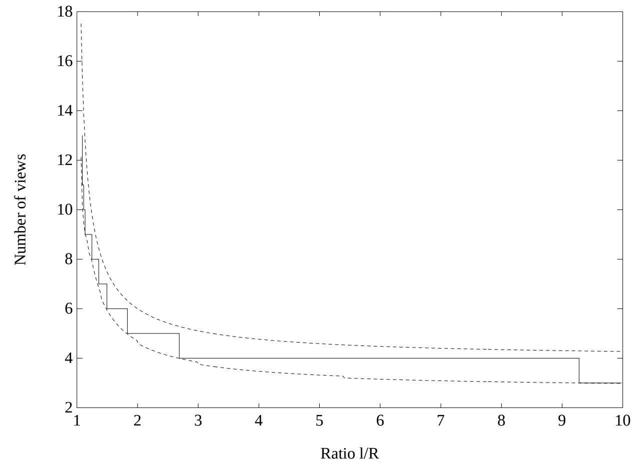

Figure 5. The necessary number of views.

4. FLATTENED CURVED SURFACE MOSAICING

Image mosaicing is employed for assembling a set of images in order to reconstruct the entire scene [3]. In the approach described in this study we reconstruct the scene painted on a curved object, by mosaicing the flattened representations. The proposed mosaicing approach is based on matching, assuming that each two images to be mosaiced have an overlapping part. The matching of the two parts is performed by comparing the graylevel values in their correspondent pixels. Distortions in these values may result in mismatch. The perspective distortion analysis presented in the previous Section provides us with a measure to evaluate the distortions resulted after flattening images of cylindrical surfaces. Let us consider that all the images to be mosaiced are taken at the same distance l from the cylinder's axis. The region from the cylindrical surface which projects in the image without significant distortion contains neighboring arcs having small size variation from each other :

$$\left| \frac{|\widehat{A_k A_{k-1}}| - |\widehat{A_{k-1} A_{k-2}}|}{|\widehat{A_m A_{m-1}}| - |\widehat{A_{m-1} A_{m-2}}|} \right| = \left| \frac{\beta_k + \beta_{k-1} - 2\beta_{k-2}}{\beta_m + \beta_{m-2} - 2\beta_{m-1}} \right| \leq \delta \tag{13}$$

where $|\widehat{A_k A_{k-1}}|$ and $|\widehat{A_{k-1} A_{k-2}}|$ are evaluated in (12), and δ is a small constant, measuring the difference in the arc length variation, representing a distortion measure. As it can be observed from Figure 3, the neighboring images are likely to be differently distorted in the overlapping regions. Let us consider that the minimal distortion condition from (13) is verified for $k = d, \ldots, 2m - d$, where d is the pixel index where we obtain the equality in (13). This interval represents the region where the curves bottom out in Figure 3. The image size that is unreliable for matching is evaluated in Figure 4 when considering m=1000. It can be observed in this figure that when $\mu = l/R$ increases, the size of the region containing geometrical distortions decreases.

Each two images must overlap on a region corresponding to an angle larger than $2\beta_d$, where we do not have big distortions according to (13). This condition provides us with a criterion to chose the minimal necessary number of images in order to mosaic the entire scene painted on the cylinder. If we consider that each pixel in the scene should be projected in two neighboring images at most, we obtain the maximum number of images. The minimal and the maximal numbers of images are :

$$\frac{\pi}{\arccos\left(\frac{R}{l}\right) - \beta_d} < n < \frac{2\pi}{\arccos\left(\frac{R}{l}\right)}, \tag{14}$$

where the angle β_d is derived from (13) and corresponds to the arc $|\widehat{A_1 A_d}|$. For an appropriate threshold δ in (13) the following condition is fulfilled :

$$\cos 2\beta_d > \frac{R}{l} \tag{15}$$

as we can observe from Figures 3 and 4. The bounds in the number of images to be taken around a cylinder are represented in Figure 5. In the same figure, the ceiling integer value of the minimum necessary number of views is marked by a continuous line. As we observe from this figure, the necessary number of images is large when the distance from the viewpoint to the cylinder's axis is small and decreases at three when the distance l is large. The position of the viewpoints with respect to the cylinder's axis can be easily derived after knowing the number of necessary images to be taken around the cylinder.

In [4] we have proposed a matching-based approach for mosaicing. The region which provides the minimum mean absolute difference, among all the possible given solutions from the search area, is chosen as the overlapping part. The search region for matching the flattened images must be larger than $2d$ pixels and should include the overlapping part. The regions which contain big perspective distortions are not appropriate to be used in the matching. After the calculation of the overlapping area, we evaluate the displacement of each image with respect to the others, and the entire scene is assembled. The output in the overlapping region is calculated as a weighted sum of the correspondent pixels from the component images [4]. The weight is proportional to the distance to the nonoverlapping parts of the respective images. The proposed procedure can be easily extended to mosaic many images having horizontal and vertical overlapping areas.

(a), (b) Painting on an arch ;

(c), (d) The flattened surface representations ;

(e) The reconstruction of the painting;

Figure 6. Reconstruction of an arch painting by mosaicing.

(a), (b), (c) Original images of a cylindrical cup ;

(d), (e), (f) The flattened surface representations ;

(g) Result of the mosaicing algorithm ;

Figure 7. The mosaic of the cylinder's flattened images.

5. SIMULATION RESULTS

We present the results provided by the proposed algorithm in two examples. In Figures 6 (a) and (b) two images representing parts of a Byzantine painting on an arch are shown. The images of the mural painting present distortions, caused by the surface of the support, that depend on the view-angle. After localizing the curved surfaces, we flatten the projected images of the paintings as presented in Figures 6 (c) and (d). The proposed algorithm based on matching was used for mosaicing the flattened images thus producing the image displayed in Figure 6 (e).

The mosaicing algorithm is applied for reconstructing the decorative scene on the surface of a cylindrical cup. The scene contains two parallels representing the margins of the decorative pattern. These parallels are used to derive the localization parameters when taking the pictures from various viewpoints, as it was described in Section 2. The radius of the cylindrical object is 100 mm and the distance from the cylinder's axis to the viewpoints is 550 mm. We can observe from Figure 5 that for $\mu = 5.5$ we need four images in order to represent the entire surface of the cylinder. Due to the cup handle and because the scene depicted does not cover all around the cup surface we use only three images for reconstructing the scene, in our experiments. They are taken from three different viewpoints such that each two images have common areas, as shown in Figures 7 (a), (b), (c). In these figures we can observe that the distortions in the cup representations due to perspective projection are larger in the tangential regions from the viewpoint than in the region of the revolution axis projection. The flattened images, after scaling, are shown in Figures 7 (d), (e), (f). For mosaicing the three flattened representations we do not take into account the regions containing severe distortions. The mosaic of the three pictures is provided in Figure 7 (g). This figure represents the flattened version of the entire scene depicted on the cup.

6. CONCLUSIONS

In this study we propose a new approach for representing the scene painted on the surface of curved objects. We describe a method for finding the localization of the curved object from one perspective view. We provide a theoretical analysis of the geometrical distortions due to the perspective projection when the images are painted on cylindrical surfaces. The results of this analysis consist of deriving the size of the image region which contains distortions. As a consequence, we derive the necessary number of views to represent the scene depicted on a cylindrical surface. We apply a matching-based mosaicing algorithm in order to estimate the displacements of the flattened surfaces. This work can be applied for the visualization of mural paintings which are painted on curved surfaces. The mosaiced image can be used in painting restoration.

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