OBJECT SEGMENTATION IN 3-D IMAGES BASED ON ALPHA-TRIMMED MEAN RADIAL BASIS FUNCTION NETWORK

Adrian G. Borş *Ioannis Pitas*

Department of Informatics, University of Thessaloniki,
Thessaloniki 54006, Greece
E-mail: {adrian,pitas}@zeus.csd.auth.gr

ABSTRACT

This paper presents a new approach for 3-D object segmentation. Objects from a stack of images are represented as overlapping ellipsoids. Graylevel statistics and shape features are simultaneously employed for object modeling in an unsupervised approach. The extension of the Hough Transform in the 3-D space is used for finding the ellipsoid centers. Each ellipsoid is modeled by a Radial Basis Function (RBF) and the entire structure is represented by means of an RBF network. The proposed algorithm is applied for blood vessel segmentation from tooth pulp in a stack of microscopy images.

1 Introduction

Representation and recognition of 3-D objects is an important task in structure identification and visualization [1]. The main approaches in 3-D object identification consist of representing each 3-D object either by a global model description or as a set of component elements. Various model-based supervised classifiers have been tested in segmenting 3-D images in [2, 3]. Each region is associated with a multivariate Gaussian mixture density in [3]. The initial Gaussian parameter estimates in [3] are obtained by means of k-means clustering. RBFs were used for 3-D iterative image reconstruction from projection data in [4].

In this paper we propose a pattern classification approach for segmenting 3-D objects. The input space consists of four features, denoting the voxel coordinates and the graylevel. The parameters of the ellipsoids can be found using the normalized first and second order moments [5]. In order to estimate the ellipsoid shape in noise we employ the α-Trimmed Mean algorithm [6]. A classical RBF and Median RBF learning algorithms described in [7] are particular cases of the proposed algorithm. The extention of the Hough Transform in 3-D is employed for estimating the centers of the ellipsoids in the context of the α-Trimmed Mean RBF training algorithm. Examples when applying the proposed algorithm in 3-D image segmentation are provided.

2 Segmentation Criterion

A classification approach is employed for 3-D object segmentation. An object is recognized according to the pattern classification theory :

$$p(\mathcal{O}_k|\mathcal{F}) = \max_{j=1}^{M} p(\mathcal{O}_j|\mathcal{F}) \quad (1)$$

where M is the total number of objects, $\mathcal{O}_k$ is the object to be identified and $\mathcal{F}$ represents a volumetric image. According to the Bayesian rule :

$$p(\mathcal{O}_j|\mathcal{F}) = \frac{p(\mathcal{F}|\mathcal{O}_j)\, p(\mathcal{O}_j)}{p(\mathcal{F})}. \quad (2)$$

We decompose the object probabilities in a set of component probabilities. The relationship (1) becomes :

$$\prod_{m=1}^{L} [p(\mathcal{F}|g_{mk})\, p(g_{mk})] = \max_{j=1}^{M} \left\{ \prod_{n=1}^{L} [p(\mathcal{F}|g_{nj})\, p(g_{nj})] \right\} \quad (3)$$

where the components making up the object $\mathcal{O}_j$ are denoted as g_{nj}. Each of the component probabilities can be expressed as an energy function which is denoted as $\phi_m(\mathbf{X})$.

The *a priori* probabilities $p(g_{nj})$ scaled by a constant are taken as weighting factors, denoted by λ_{nj}. The relationship (3) can be expressed as :

$$\sum_{m=1}^{L} \lambda_{mk} \exp[-\phi_m(\mathbf{X})] = \max_{j=1}^{M} \left\{ \sum_{n=1}^{L} \lambda_{nj} \exp[-\phi_n(\mathbf{X})] \right\}. \quad (4)$$

This structure is implemented in an RBF network.

3 3-D Hough Transform

We can observe that a Gaussian function describes geometrically an ellipsoid. Each continous object is considered as made up from a set of ellipsoids. We extend the Hough Transform (HT) in the 3-D space for identifying the centers of the ellipsoids. HT represents a mapping from the image features to sets of points in a parameter space [5]. In order to find the object edges we apply the 3-D extension of the Sobel edge detector algorithm which provides the edge orientation as well. We consider the spherical coordinate system which appropriately matches the ellipsoid description. At the intersections of perpendiculars on the 3-D edges that are raised at the points where a significant edge is located we increment an accumulator :

$$x = x_e + r\cos(\beta)\cos(\theta) \tag{5}$$

$$y = y_e + r\cos(\beta)\sin(\theta) \tag{6}$$

$$z = z_e + r\sin(\beta) \tag{7}$$

for a certain assumed interval $r = (0, R)$ starting with the position of the edge points (x_e, y_e, z_e), where the edge orientation is given by (β, θ). Each selected edge point will generate a different surface in the parameter space, but all surfaces generated by the same model instance will intersect at a common point which describes the model instance. The bigger accumulator represents a larger probability that the corresponding location represents the center of an ellipsoid.

4 Alpha-Trimmed Mean RBF Network

RBF network consists of a two-layer neural network. A hidden-layer unit implements the Gaussian function :

$$\exp[-\phi_m(\mathbf{X})] = \exp[-(\mathbf{X}-\mu_m)^T \mathbf{\Sigma}_m^{-1}(\mathbf{X}-\mu_m)] \tag{8}$$

where μ_m denotes the center vector and $\mathbf{\Sigma}_m$ the covariance matrix. The output of the network for the data vector $\mathbf{X}$ is denoted by $Y_k(\mathbf{X})$, where $k = 1, \ldots, N$ and is limited to the interval $(0, 1)$ by a sigmoidal function :

$$Y_k(\mathbf{X}) = \frac{1}{1 + \exp\left(-\sum_{m=1}^{L} \lambda_{mk} \exp[-\phi_m(\mathbf{X})]\right)} \tag{9}$$

where L is the total number of hidden units. Each output corresponds to an object in the image.

In the training algorithm we generate a set of centers at random and the algorithm calculates the Euclidean distance from a data sample to each of them. The closest center to the given data vector is chosen to be updated :

$$||\mathbf{X} - \hat{\mu}_k|| = \min_{i=1}^{L} ||\mathbf{X} - \hat{\mu}_i||. \tag{10}$$

In the classical approach based on the Learning Vector Quantization, the center is updated using :

$$\hat{\mu}_k = \hat{\mu}_k + \frac{1}{N_k}(\mathbf{X} - \hat{\mu}_k) \tag{11}$$

where N_k is the number of data samples associated with the kth basis function. For the covariance matrix, classical estimate can be used :

$$\hat{\mathbf{\Sigma}}_k = \frac{\sum_{i=0}^{N_k} (\mathbf{X}_i - \hat{\mu}_k)\,(\mathbf{X}_i - \hat{\mu}_k)^T}{N_k - 1} \tag{12}$$

In [7] the marginal median and median of the absolute deviations from the median estimators were employed for estimating the RBF center and covariance matrix. In this study, after ordering the data samples $X_{(0)} < \ldots < X_{(N_k)}$, we use the α-Trimmed Mean algorithm :

$$\begin{array}{lll} \hat{\mu}_k(t) = & \hat{\mu}_k(t-1) & + \dfrac{c_i\,(\mathbf{X}_{(i)} - \hat{\mu}_k(t-1))}{\sum_{i=\alpha_k N_k}^{N_k - \alpha_k N_k} c_i} \\ & \text{if} & \alpha_k N_k < i < N_k - \alpha_k N_k \\ \hat{\mu}_k(t) = & \hat{\mu}_k(t-1) & \text{otherwise} \end{array} \tag{13}$$

where c_i is the accumulator provided by the 3-D Hough Transform used as a weight and α_k is the percentage to be trimmed. It has been proved that by trimming an ellipse, its center estimation is unbiased [8].

The parameter α_k is chosen according to the data distribution. The following measure is used for estimating the tail of the data distribution [9, 10] :

$$Q = \frac{U[0.5] - L[0.5]}{U[0.05] - L[0.05]} \tag{14}$$

where $U[\beta]$, $L[\beta]$ represent the average of the upper and respectively the lower β percentage of data samples assigned to a specific basis function. The number of data samples to be trimmed away relies directly on the value of Q :

$$\hat{\alpha}_k = \frac{1-Q}{2}. \tag{15}$$

When the distribution is long tailed, the amount of data samples to be trimmed is large. When the distribution tail is short, the amount of data samples to be trimmed is small.

In order to estimate the covariance matrix, we firstly order the data samples with respect to their Mahalanobis distance to the center vector and we trim an α_M percentage of the data situated at the higher extreme of the resulting distribution. The covariance matrix can be estimated using :

$$\hat{\mathbf{\Sigma}}_k = \frac{\sum_{i=0}^{N_k - 2\alpha_{k,M} N_k} (\mathbf{X}_{(i),M} - \hat{\mu}_k)\,(\mathbf{X}_{(i),M} - \hat{\mu}_k)^T}{(1-\alpha_{k,M})(N_k - 2\alpha_{k,M} N_k)} \tag{16}$$

where the division by the factor $(1 - \alpha_{k,M})$ is used in order to compensate for the data loss in the case of ideal ellipsoids [8]. The percentage of the data samples to be trimmed for estimating the covariance matrix is calculated using :

$$\hat{\alpha}_{k,M} = 1 - \frac{U[0.5]}{U[0.05]} \tag{17}$$

We can observe that for $\alpha_k, \alpha_{k,M} = 0$ in (13,16) we obtain the classical RBF training algorithm and for $\alpha_k = 0.5$ we obtain the Median RBF algorithm [7].

In order to group various ellipsoids in distinct objects we employ two criteria: the first criterion considers the compactness and the second takes into account the similarity in the graylevel statistics. Each voxel in the volumetric image is assigned to a basis function. For the regions from the volumetric image which are neighboring each other we estimate a graylevel similarity and we provide a label to each of them, denoted as $F_k(\mathbf{X})$:

$$\text{If } \sum_{b=0}^{255} \left| \frac{g_m(b)}{N_m} - \frac{g_n(b)}{N_n} \right| < h$$

$$\text{then } \left\{ \begin{array}{l} F_k(\mathbf{X}) = 1 \\ F_j(\mathbf{X}) = 0 \end{array} \right. \forall\, j \neq k$$

for all the voxels associated with the hidden units m and n, where h is a threshold, $F_k(\mathbf{X})$ is the decision function for the class k and $g_m(b)$ represents the histogram of the object component m. In this way, the unsupervised problem is transformed in a self-supervised one. Based on these labels and on the network outputs as provided by (9) we can use the backpropagation algorithm in order to estimate the output weights λ_{mk} [7].

5 Simulation Results

The proposed algorithm has been tested on a stack of 60 microscopy images, representing the internal structure of a tooth pulp. A frame is shown in Figure 1 (a). A rendered 3-D view of the stack of images is displayed in Figure 2 (a). We intend to segment the blood vessels represented as continuous dark areas. As it can be observed in these images, the tissue structure is very noisy and the objects are not well defined. In the training stage we have used only 20 frames (one out of each three consecutive frames) and the features were extracted from 16×16 pixel blocks. This ensured a great reduction in the number of input data samples and, implicitly, in the training time. After estimating the parameters from the given subset of data, we apply the model to the entire stack of images, at pixel-resolution. The classical RBF algorithm has been compared with the α-Trimmed Mean RBF. The segmentation results of the frame from Figure 1 (a), when using classical RBF as provided by (11,12) and α-Trimmed Mean RBF (13,16) algorithms are shown in Figures 1 (b) and (c), respectively. We can observe from these figures that α-Trimmed Mean RBF provides better segmentation results than the classical training algorithm. The training is unsupervised in both algorithms. The α-trimmed mean algorithm elliminates most of the noise while the joint geometrical and graylevel features contributes to a better segmentation of the objects. The result of the 3-D segmentation is visualized for various perspective angles in Figures 2 (b) and (c). From these figures it can be observed that the 3-D object modeling and segmentation is quite accurate.

6 Conclusions

We propose a new algorithm for modeling and segmenting objects in 3-D images. A classification approach is employed for the 3-D segmentation. The objects are considered as composed of overlapping ellipsoids. The classifier employed in modeling the 3-D structure as well as the graylevel is the RBF network, where each basis unit corresponds to an ellipsoid. We provide a robust learning algorithm for the RBF network based on the α-Trimmed Mean statistics. In order to find the centers of the ellipsoids we develop a 3-D Hough Transform which is integrated in the training algorithm. The proposed algorithm was successfully applied in a stack of noisy microscopy images.

References

[1] G. Lohman, *Volumetric image analysis.* J. Wiley-Teubner, 1998.

[2] M. Kamber, R. Shinghal, D. L. Collins, G. S. Francis, A. C. Evans, "Model-based 3-D segmentation of multiple sclerosis lesions in magnetic resonance brain images," *IEEE Trans. on Medical Imaging,* vol. 14, no. 3, pp. 442-453, Sep. 1995.

[3] A. Lundervold, G. Storvik, "Segmentation of brain parenchyma and cerebrospinal fluid in multispectral magnetic resonance images," *IEEE Trans. on Medical Imaging,* vol. 14, no. 2, pp. 339-349, June 1995.

[4] S. Matej, R. M. Lewitt, "Practical considerations for 3-D image reconstruction using spherically symmetric volume elements," *IEEE Trans. on Medical Imaging,* vol. 15, no. 1, pp. 68-78, 1996.

[5] R.M. Haralick, L.G. Shapiro, *Computer and Robot Vision, vol. I.* Reading, MA:Addison-Wesley, 1992.

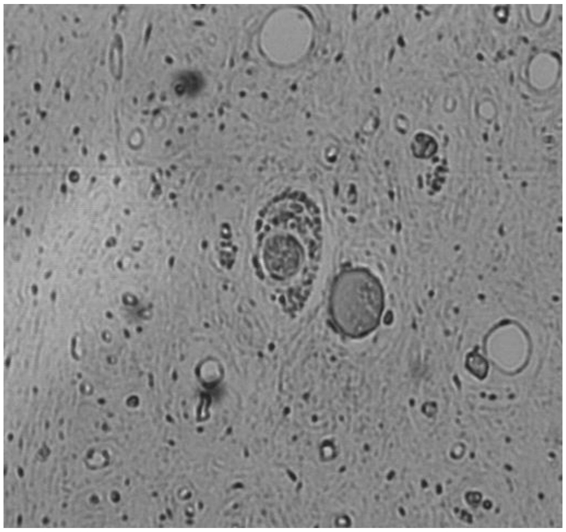
[6] G. Seber, *Multivariate Observations.* Wiley, 1986.

[7] A. G. Borş, I. Pitas, "Median Radial Basis Function neural network," *IEEE Trans. on Neural Networks,* vol. 7, no. 6, pp. 1351-1364, 1996.

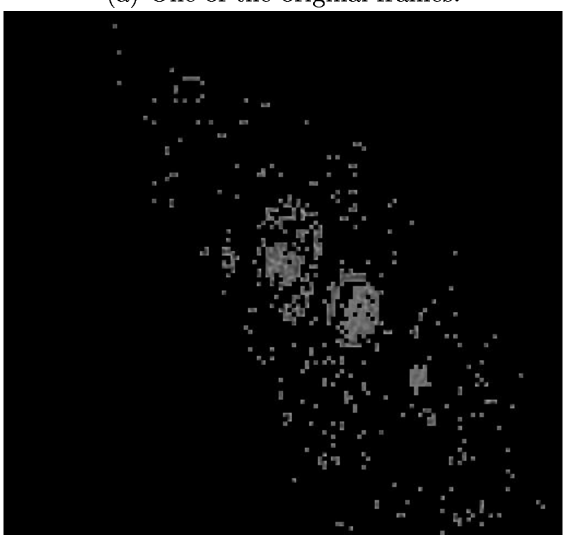
[8] A. G. Bors, I. Pitas, "Alpha-trimmed Mean Radial Basis Functions and their Application in Object Modeling," *CD-ROM Proc of the IEEE Workshop on Nonlinear Signal and Image Processing (NSIP'97),* Michigan, USA, 7-11 Sept. 1997.

[9] P. Prescott, "Selection of trimming proportions for robust adaptive trimmed means," *J. Am. Stat. Assoc.,* no. 361, vol. 73, pp. 133-140, 1978.

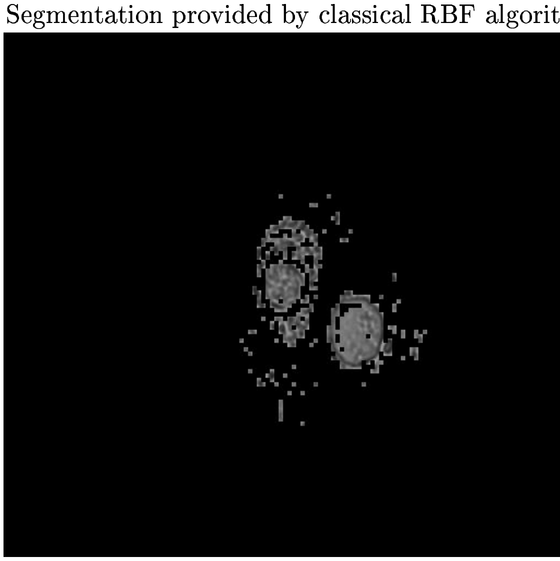
[10] D. M. Titterington, "Estimation of correlation coefficients by ellipsoidal trimming," *Appl. Stat.,* no. 3, vol. 27, pp. 227-234, 1978.



(a) One of the original frames.

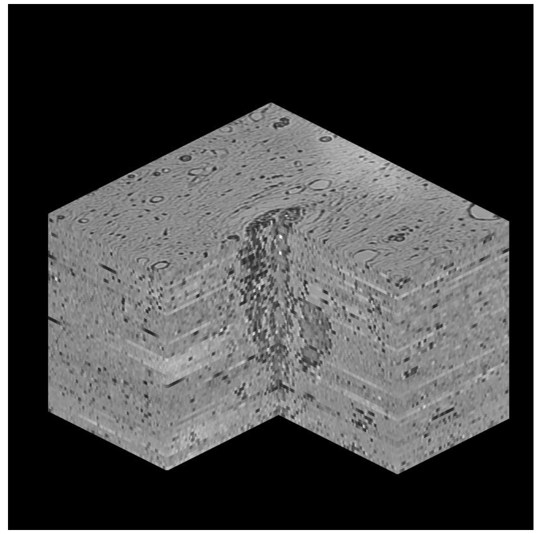


(b) Segmentation provided by classical RBF algorithm.

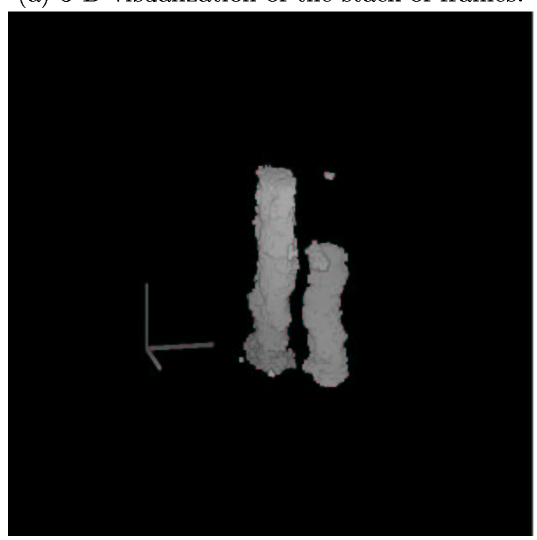


(c) Segmentation provided by the α-Trimmed Mean RBF algorithm.

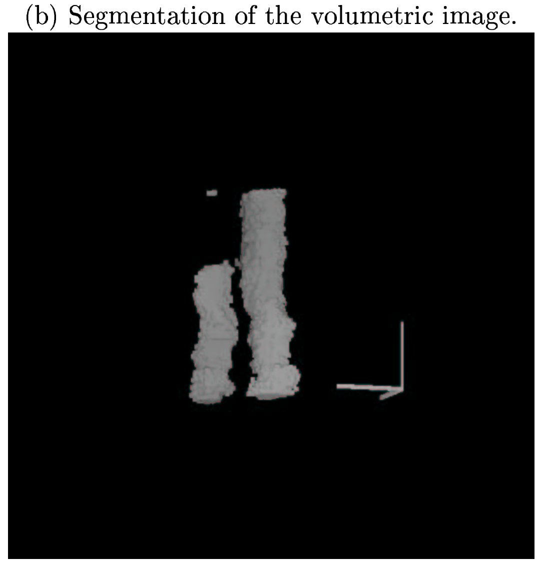
Figure 1: Frame from a stack of microscopy images.



(a) 3-D visualization of the stack of frames.



(b) Segmentation of the volumetric image.



(c) Blood vessel visualization.

Figure 2: 3-D representation of a volumetric image and its segmentation.