NON-LINEAR WATERMARK DETECTOR EMBEDDED IN FOURIER DOMAIN

V. Solachidis and I.Pitas

Department of Informatics,University of Thessaloniki
Thessaloniki 541 24, Greece, Tel,Fax:+30-2310996304
e-mail: vasilis, pitas@zeus.csd.auth.gr

ABSTRACT

This paper deals with the statistical analysis of the behavior of a blind copyright protection watermarking system based on pseudorandom signals embedded in the magnitude of the Fourier transform of the host data. The data can be either one-dimensional (sound), two dimensional (image) or three dimensional (video, 3d volumes). The detector scheme does not depend on the host data dimension. The analysis performed involves theoretical evaluation of the statistics of the Fourier coefficients and an optimum detector design both for additive and multiplicative embedding. It is proved that the optimum detector is not the widely used correlator one. Finally, experimental results are presented in order to show the proposed detector's efficiency versus that of the correlator detector.

1. INTRODUCTION

The development of the digital services eased the illegal copying, reproduction and distribution of copyright material. Therefore, new requirements have emerged for multimedia security and copyright protection techniques. Watermarking has been proposed as an efficient tool for copyright protection. The related research has exhibited tremendous growth in the past decade.

This paper deals with designing of an optimal detector for transform watermarking methods. The watermark embedding is performed in the magnitude of the Fourier transform domain of the host data. It should be noted that the data may be one dimensional (sound), two dimensional (image) or three dimensional (video, 3d volumes). Also, the detector scheme does not depend on the data dimension that has to be watermarked. We investigate both watermarking (the additive and the multiplicative) methods. The host data can be real [1] or complex [2].

The paper is organized as follows. The watermark design and embedding algorithm is described in the Section 2. In section 3 the detection procedure and the optimum detector design are presented. In the next section experimental results are shown, which depict the efficiency of the proposed detection versus that of the correlator detector. Finally, in section 5 conclusions and future work are presented.

2. WATERMARK EMBEDDING

Let $x(\mathbf{i})$ be the original data where $x(\mathbf{i})$ is an D-dimensional complex signal ($x(\mathbf{i}) \in \mathbb{C}^D$), $x(\mathbf{i}) = x_R(\mathbf{i}) + i\ x_I(\mathbf{i})$ and $\mathbf{i} \in [0, N_1-1] \times [0, N_2-1] \times ... \times [0, N_D-1] = \mathbf{N}$. Let also $X(\mathbf{i})$ be the Fourier transform of $x(\mathbf{i})$ and $M(\mathbf{i})$ and $P(\mathbf{i})$ are the magnitude of the Fourier transform ($M(\mathbf{i}) = |X(\mathbf{i})|$) and its phase respectively. Let $X_R(\mathbf{k})$ and $X_I(\mathbf{k})$ denote the real and the imaginary part of $X(\mathbf{k})$ respectively. Let also $W(\mathbf{i})$ denote the watermark sequence. We assume that $W(\mathbf{i})$ consists solely of 1 and -1 that are uniformly distributed.

The watermark should be embedded in the middle frequencies due to the following reasons. Modifications in the low frequencies of the Fourier transform will cause perceptible changes in the spatial domain. Furthermore, compression affects mostly the high frequencies of the Fourier transform. Thus, the watermark should be added in the middle frequency range because, if carefully designed, it will be robust against compression and at the same time imperceptible. Considering that the zero frequency term (DC term) is in the center of the transform domain, the watermark is embedded in a ring that covers the middle frequencies.

$$W(\mathbf{i}) = \begin{cases} 0, & \text{if } r < R_1 \text{ or } r > R_2 \\ \pm 1, & \text{if } R_1 < r < R_2, \end{cases} \tag{1}$$

where r is the Euclidean distance of $\mathbf{i}$ from the DC term. Taking into account that the DC term satisfies $DC = M(\mathbf{N/2})$ then $r = ||\mathbf{N/2} - \mathbf{i}||$.

The watermark can be embedded in an additive or multiplicative way. Let $M'(\mathbf{i})$ be the watermarked data. Thus, in case of additive embedding $M'(\mathbf{i})$ equals:

$$M'(\mathbf{i}) = M(\mathbf{i}) + p\, W(\mathbf{i}). \tag{2}$$

In case of multiplicative it equals to:

$$M'(\mathbf{i}) = M(\mathbf{i}) + p\, W(\mathbf{i}) M(\mathbf{i}) = M(\mathbf{i}) \left(1 + p\, W(\mathbf{i})\right). \tag{3}$$

3. WATERMARK DETECTION

In this section the optimal watermark detector will be evaluated and the probability density function (pdf) of the magnitude of the DFT coefficients will be estimated. Then, the distribution parameter will be evaluated and finally the optimum detector will be estimated by using the likelihood ratio test (LTR). The above procedure will be performed for both additive and multiplicative embedded watermarks.

3.1. Probability density function of magnitude of DFT coefficients

In order to extract the optimum detector, the probability density function of the magnitude for the DFT coefficients has to be estimated. We assume that the samples of the host signal can be modeled as independent identically distributed (i.i.d) random variables obeying a distribution function $f_{\mathbf{x}}(\mathbf{x})$ with the following properties:

$$E(x_i) = \mu_x, \ \forall i \in \mathbf{N}$$

$$E(x_i x_{i+k}) = 0, \ \forall i \in \mathbf{N}.$$

In the sequel, the distribution of each coefficient of $X(\mathbf{i})$ will be calculated. Assuming that some of the $N_1, N_2, ..., N_D$ (at least one) are greater than 30, we can conclude, according the Central Limit Theorem, that the distribution of each DFT coefficient is Gaussian. The statistics for the Fourier transform coefficients are given by:

$$E(X(\mathbf{k})) = \begin{cases} 0, & \mathbf{k} \neq \mathbf{0} \\ [E(x_R) + i\ E(x_I)] \prod_{i=1}^{N_D} N_i, & \mathbf{k} = \mathbf{0} \end{cases} \tag{4}$$

$$\mathrm{var}(X_R(\mathbf{k})) = \begin{cases} \prod_{i=1}^{D} N_i \frac{\mathrm{var}(x_R)+\mathrm{var}(x_I)}{2}, & \mathbf{k} \neq \mathbf{0}, \mathbf{1} \cdot \frac{N}{2} \\ \prod_{i=1}^{D} N_i \mathrm{var}(x_R), & \mathbf{k} = \mathbf{0}, \mathbf{1} \cdot \frac{N}{2} \end{cases} \tag{5}$$

$$\mathrm{var}(X_I(\mathbf{k})) = \begin{cases} \prod_{i=1}^{D} N_i \frac{\mathrm{var}(x_R)+\mathrm{var}(x_I)}{2}, & \mathbf{k} \neq \mathbf{0}, \mathbf{1} \cdot \frac{N}{2} \\ \prod_{i=1}^{D} N_i \mathrm{var}(x_I), & \mathbf{k} = \mathbf{0}, \mathbf{1} \cdot \frac{N}{2} \end{cases} \tag{6}$$

where $E(\cdot)$ is the expected value and $var(\cdot)$ the variance. A detailed prof of the above equations is given in the Appendix. Equation (4) shows that the mean of all the coefficients except the DC term equals zero. According to (5-6) the variances of the real and the imaginary part for almost all the coefficients are equal. Thus, both the real and the imaginary part of the Fourier coefficients obeys the same Gaussian distributions with zero mean ($X_R(\mathbf{k}) \sim N(0, \sigma^2)$ and $X_I(\mathbf{k}) \sim N(0, \sigma^2)$). Therefore, the pdf of $|X(\mathbf{k})| = \sqrt{X_R^2(\mathbf{k}) + X_I^2(\mathbf{k})}$ follows the Rayleigh distribution [3]:

$$|X_i(\mathbf{k})| \sim f_x(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right),\ x > 0.$$

The above, according the equations (5-6) stands for every coefficient of the magnitude except the DC term $|M(\mathbf{0})|$ and the middle term $|M(N/2, N/2, ..., N/2)|$.

Rayleigh distribution is a member of the exponential family. It can be easily proved that $\sigma = \sqrt{\frac{\sum_{i=1}^{N} X_i^2}{2N}}$ is a minimum variance bound estimator (its variance reaches the lower bound of the Cramer-Rao inequality).

3.2. Likelihood ratio test (LRT)

Since the probability density function of the distribution of the coefficients is known, an optimum detector will be computed using the likelihood ratio test (LRT). According to the Neyman-Pearson theorem [3] and in order for the probability of detection to be maximized the following quantity has to be computed:

$$L(x) = \frac{p(\mathbf{x}; H_1)}{p(\mathbf{x}; H_0)}. \tag{7}$$

In the sequel the probability density function of the watermarked signal will be computed for both the watermarked with a known and an unknown watermark. Due to the fact that the watermark consists of ± 1 and its mean value is zero then its pdf is:

$$f_w(x) = \begin{cases} 0.5 & x = 1 \\ 0.5 & x = -1 \\ 0 & otherwise \end{cases} \tag{8}$$

The two hypotheses that will be examined are:
H_0: data is not watermarked or watermarked by another (unknown) watermark W'
H_1: data is watermarked by watermark W
In the case of an additive watermark, it can be easily proved that the pdf of the watermarked signal is equal with [4]

$$s(x) \sim \frac{1}{2}[f_x(x-p) + f(x+p)] \tag{9}$$

By substituting f_x by the probability density function of Rayleigh distribution in the above equation, we result in:

$$s(x) \sim \frac{1}{2}\left[\frac{x-p}{\sigma^2} e^{\frac{-(x-p)^2}{2\sigma^2}} + \frac{x+p}{\sigma^2} e^{\frac{-(x+p)^2}{2\sigma^2}}\right]. \tag{10}$$

In case of multiplicative watermarking the pdf has the form:

$$s(x) \sim \frac{1}{2}\left[\frac{1}{1+p}\, f_x\left(\frac{x}{1+p}\right) + \frac{1}{1-p}\, f_x\left(\frac{x}{1-p}\right)\right]$$

and after the substitution we derive to

$$s(x) \sim \frac{1}{2}\left[\frac{1}{1+p}\ \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2(1+p)^2}\right) + \frac{1}{1-p}\ \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2(1-p)^2}\right)\right] \tag{11}$$

Thus, using the above equations both probabilities $p(x[n], H_0)$, $p(x[n], H_1)$ can be calculated. In the case of hypothesis H_0, for each $s[n]$, the probability of $W[n] = 1$ equals the probability of $W[n] = -1$, $(PW[n] = 1 = PW[n] = -1 = 1/2)$.

Therefore, the above equations (10,11) give the probabilities for hypothesis H_0. Hence for the additive case,

$$p(x[n];H_0) \sim \frac{1}{2}\left[\frac{x[n]-p}{\sigma^2}\exp\left(-\frac{(x[n]-p)^2}{2\sigma^2}\right) + \frac{x[n]+p}{\sigma^2}\exp\left(-\frac{(x[n]+p)^2}{2\sigma^2}\right)\right] \tag{12}$$

and for the multiplicative case:

$$p(x[n];H_0) \sim \frac{1}{2}\left[\frac{1}{1+p}\,\frac{x[n]}{\sigma^2}\exp\left(-\frac{x[n]^2}{2\sigma^2(1+p)^2}\right) + \frac{1}{1-p}\,\frac{x[n]}{\sigma^2}\exp\left(-\frac{x[n]^2}{2\sigma^2(1-p)^2}\right)\right] \tag{13}$$

In the case of hypothesis H_1, the signal is watermarked by the known watermark W. Thus, the probabilities are:

$$p(x[n];H_1) \sim \frac{x[n]+w[n]p}{\sigma^2}\exp\left(-\frac{(x[n]+w[n]p)^2}{2\sigma^2}\right) \tag{14}$$

for the additive case and:

$$p(x[n];H_1) \sim \frac{x[n]}{\sigma^2(1+w[n]p)^2}\exp\left(-\frac{x[n]^2}{2\sigma^2(1+w[n]p)^2}\right) \tag{15}$$

for the multiplicative case. Assuming independence between the samples of X, we conclude that:

$$p(\mathbf{x};H_j) = \prod_{i=0}^{n-1} p(x[i];H_j) \quad ,j=0,1 \tag{16}$$

Using equations 12,13 ,14, 15, equation 7 has the form

$$L(x) = \frac{p(\mathbf{x};H_1)}{p(\mathbf{x};H_0)} \tag{17}$$

$$L(x) = \frac{\prod_{i=0}^{N-1} 2(x[i]+w[i]p)}{\prod_{i=0}^{N-1}\sum_{j=0}^{1}\left[(x[i]-(-1)^j p)\exp\left[\frac{x[i]p(w[i]+(-1)^j)}{\sigma^2}\right]\right]} \tag{18}$$

$$L(x) = \prod_{i=0}^{N-1} \frac{2}{\sum_{i=0}^{1}\left[\frac{(1+w[i]p)^2}{\left(1+(-1)^j p\right)^2}\exp\left(\frac{px[i]((-1)^j - w[i])}{\sigma^2(1+p)^2(1+w[i]p)^2}\right)\right]} \tag{19}$$

for the additive and multiplicative case respectively.

4. EXPERIMENTAL RESULTS

In this section experiments are performed in order to verify the superiority of the proposed detector against the correlator. The experiments have been performed on digital images. The gray scale (512×512) Lena image was used as a host image. A random watermark was embedded in it. The embedding procedure has been performed on either additively or multiplicatively. Then, two detector outputs have been calculated using the proposed detector and the correlator. In order to calculate both false alarm and false rejection probabilities both correct and erroneous keys have been used during detection. The watermarked image was altered by intentional attacks for the examination of the robustness against attacks. In Figure 1 the ROC curves are shown both for the proposed detector and correlator. From the above figure, can be seen very clear the superiority of the proposed detector is against detector.

Figure 1: ROC curves for both detectors (multiplicative embedding)

5. CONCLUSIONS AND FUTURE WORK

This paper deals with the statistical analysis of the behavior of a blind copyright protection watermarking system based on pseudorandom signals embedded in the magnitude of the Fourier transform of the data and the designing of an optimum detector. The detector scheme does not depend on the data dimension that has to be watermarked thus the data can be either one, two or three dimensional. Both additive and the multiplicative embedding methods are examined and experimental results are performed in order to show the detector's efficiency against correlator. Data following more realistic models than i.i.d. are under consideration for future work.

REFERENCES

[1] V.Solachidis and I.Pitas, "Circularly symmetric watermark embeddong in 2-d dft domain," *IEEE Transactions on Image Processing*, vol. 10, no. 11, pp. 1741–1753, 2001.

[2] V. Solachidis, N. Nikolaidis, and I. Pitas, "Fourier descriptors watermarking of vector graphic images," in *Proc. of ICIP'00*, Vancouver, Canada, 10-13 September 2000, vol. 3, pp. 9 – 12.

[3] S.M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice Hall, NJ, USA, 1998.

[4] A. Papoulis, *Probability & Statistics*, Prentice Hall, 1991.