Curved Surface Reconstruction Based on Parallels

William Puech, Jean-Marc Chassery and Ioannis Pitas

Abstract. In this paper we present a new approach for reconstructing images mapped or painted on curved surfaces. In the case of monocular vision, by using a priori knowledge about the support surface of the pictures and projected parallels in the image, first we derive the surface localization in the camera coordinate system. Next, we reconstruct the surface using the information of parallel curves in order to backproject the image on the surface. Finally, we simulate a camera to obtain new images from different viewpoints.

§1. Introduction

A depicted scene may be represented on curved surfaces, as shown in Figure 1. Our reconstruction method will be limited to surfaces with curvature different than zero in only one direction. The curves drawn on the scene of constant altitude in the coordinate system of the surface will be called parallels. In case of monocular vision, we need to use certain a priori knowledge to perform the 3D reconstruction of a surface. So, we use the information given by the projections of at least two parallels contained on the scene. Before reconstructing the curved surface it is necessary to locate this surface in the camera coordinate system. Next, we shall present the method of surface reconstruction using the information of the parallel curves.

An overview on the general domain of 3D reconstruction is found in the papers [4,8]. Based on the perspective projection of the curved surface and parallels on the image plane, in Section 2 we shall present briefly the method of localization by finding the projection of the revolution axis in the image [6,7]. In Section 3, we describe with more details the method of reconstruction. We use the two axes found in the localization method to backproject the parallel curves in the space with a given scale factor. In our case, the equation of the surface is derived by interpolation based on the least mean squares method. With the equation of the curved surface and its localization in the camera coordinate system, the image is backprojected on the surface in order to obtain

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Fig. 1. View of an arch of a Byzantine church.

the 3D representation of the scene. In Section 4, simulation results will first be illustrated on synthetic images, and then on an image of a mural painting shown in Figure 1. This scene could be visualized from various viewpoints.

§2. Localization

In this section we show how to locate a curved surface in the camera coordinate system from a single perspective view, as shown in Figure 2. First we want to find the projection of the revolution axis in the image. Other work has already been done on this topic, finding local symmetries [5] or by using a method based on expectation-maximization [3].

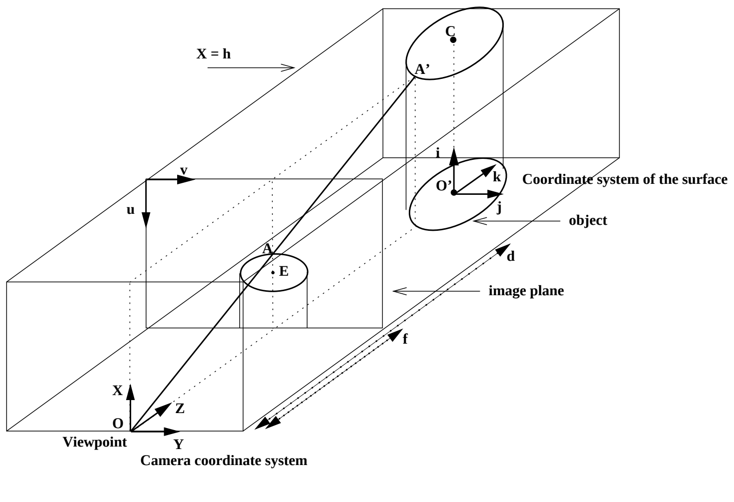

Fig. 2. A circle from a cylinder projected in the image plane.

Our approach is based on the perspective projection of the curved surface on the image as described in [7]. We identify first the common normal P_1P_2 of two curves, as shown in Figure 3(a). The slope of the straight line P_1P_2 gives us the direction of the revolution axis. In the coordinate system of the image, the equation of this axis is

$$v = A_1.u + B_1, \tag{1}$$

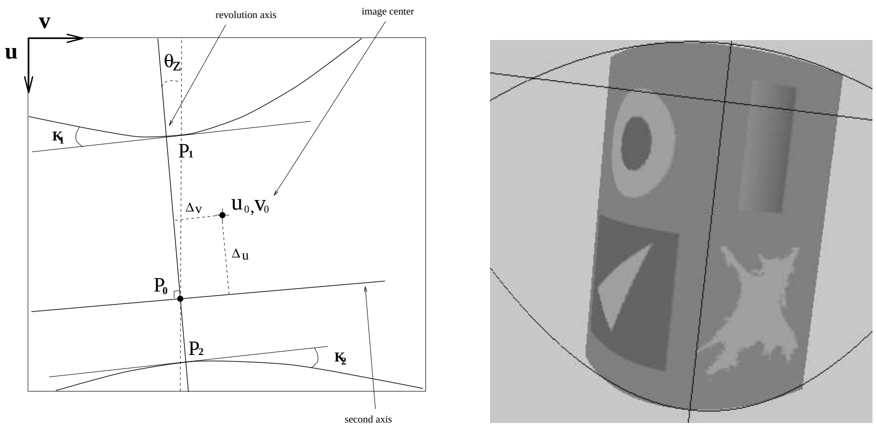

Fig. 3. (a) The revolution and the second axes detected from two curves, (b) The two axes detected on an image mapped on a cylinder.

where A_1 and B_1 are the coefficients of the straight line determining this axis. The second axis is defined as the intersection of the image plane with the plane perpendicular to the revolution axis containing the viewpoint [6]. We define P_0 as the intersection between the revolution axis and the second axis, as shown in Figure 3(a). From equation (1), we obtain the equation of the second axis:

$$v = -\frac{1}{A_1}.u + (v_0 + \frac{u_0}{A_1}). \tag{2}$$

In order to locate the object in the camera coordinate system we want to match the two vectors $\vec{x}$ and $\vec{i}$, shown in Figure 2. As described in [6] we obtain the three rotation angles:

$$\begin{cases} \theta_x &= \arctan(\frac{\Delta_v}{f.k}) \\ \theta_y &= \arctan(\frac{\Delta_u}{f.k}) \\ \theta_z &= \arctan A_1, \end{cases} \tag{3}$$

where f is the focal distance, k the resolution factor, and (Δ_u, Δ_v) the vector translation between the image center and P_0, as described in Figure 3(a). A result of the localization is shown in the Figure 3(b).

§3. 3D Reconstruction

The proposed 3D reconstruction method is based on the backprojection of two parallel curves. In Section 3.1 we analyse the curvature evolution of a cylinder when it is projected on the image plane. In Section 3.2 we explain how to reconstruct a curved surface based on the detection of parallel curves approximated by parabolic curves and ellipses.

3.1 Evolution of the curvature

When parallel circles from a cylinder are projected on the image plane ellipses with different parameters are obtained. We want to analyse the curvature of points belonging to the common normal as described Section 2. In the camera coordinate system shown in Figure 2, a cylinder is given by

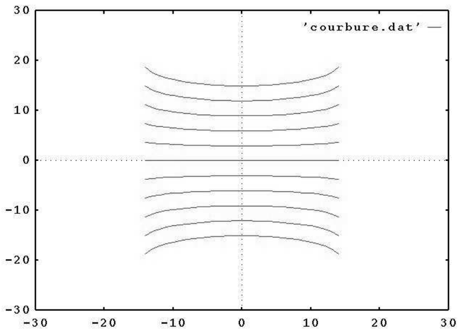

Fig. 4. Evolution of the curvature K, with $R = 100mm$, $f = 60mm$, $d = 500mm$ and $H = 250mm$ for $x = \frac{-5H}{5}, \frac{-4H}{5}, ..., \frac{4H}{5}, \frac{5H}{5}$.

$$\begin{cases} (z - z_c)^2 + y^2 = R^2 \\ -H \le x \le H, \end{cases} \tag{4}$$

where R is the radius, $2H$ the height of the cylinder and $z_c = d + R$, where d is the minimal distance between the viewpoint and the cylinder. If we take a value h for x, we obtain one circle $\mathcal{C}$ belonging to the cylinder. Let us denote the cone passing through $\mathcal{C}$ and the viewpoint O:

$$(\frac{zh}{x} - z_c)^2 + \frac{y^2h^2}{x^2} = R^2. \tag{5}$$

The intersection of this cone described in (5) and the image plane $z = f$ gives us the equation of an ellipse:

$$\begin{cases} \frac{(x-x_e)^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = f, \end{cases} \tag{6}$$

where $a = \frac{Rfh}{d^2+2Rd}$, $b = \frac{Rf}{\sqrt{d^2+2Rd}}$ and $x_e = \frac{fh(d+R)}{d^2+2Rd}$ are the three parameters of this ellipse. The curvature K for the point $A(x_a, 0, f)$, where $x_a = x_e + a$, belonging to the ellipse and to the projection of the revolution axis is

$$K = \frac{a}{b^2}. \tag{7}$$

Equations (6) and (7) prove that the curvature depends of the altitude h of the circle in the space. We obtain two linear relationships:

$$\begin{cases} K = \frac{h}{Rf} = \frac{x_a d}{Rf^2} \\ x_a = \frac{fh}{d} = x_e + a, \end{cases} \tag{8}$$

with $-H \le h \le H$, and x_a the first coordinate of A. Figure 4 illustrates the curvature evolution for different h.

3.2 Curved surface reconstruction

In this section we explain how to reconstruct a curved surface based on the detection of parallel curves used in the Section 2. Two parallel curves back-projected in the space should be mutually parallel and parallel to the plane $(y, 0, z)$. We can match the intersection P_0 of the two axes with the center of a virtual image in order to obtain a vertical position of the revolution axis [6].

After an edge detection, we should approximate the interesting curves by parabolic curves and ellipses. The equation of a parabolic curve in the image plane is

$$\begin{cases} x = ay^2 + b \\ z = f, \end{cases} \tag{9}$$

where z is the focal distance. The equation of the surface passing through the parabola and the viewpoint O, as shown in Figure 2, is

$$x = \frac{fa}{z}y^2 + \frac{z}{f}b. \tag{10}$$

The projection of this parabola in the space belongs to the surface (10) and to the plane $x = h$. The intersection of the equation (10) with the plane $x = h$ gives us the equation of an ellipse:

$$\begin{cases} \frac{\left(z - \frac{fh}{2b}\right)^2}{\left(\frac{fh}{2b}\right)^2} + \frac{y^2}{\frac{h^2}{4ab}} = 1 \\ x = h. \end{cases} \tag{11}$$

With $h = \frac{bd}{f}$ and $-H \leq x \leq H$, we obtain an object of revolution:

$$\begin{cases} \frac{\left(z - \frac{d}{2}\right)^2}{\left(\frac{d}{2}\right)^2} + \frac{y^2}{\frac{bd^2}{4af^2}} = 1 \\ -H \leq x \leq H. \end{cases} \tag{12}$$

Finally, to backproject our image on the curved surface we need only the half of the ellipse ($\frac{d}{2} \leq z$):

$$\begin{cases} z = \frac{d}{2}\left(1 + \sqrt{1 - \frac{4afy^2}{bd^2}}\right) \\ -H \leq x \leq H. \end{cases} \tag{13}$$

Now we present the method of 3D reconstruction when the curves in the image are approximated by ellipses. As shown in Figure 2, the equation of the ellipse in the image is

$$\begin{cases} \frac{(x - x_e)^2}{a^2} + \frac{y^2}{b^2} = 1 \\ z = f. \end{cases} \tag{14}$$

Thus, the equation of the surface passing through the ellipse and the viewpoint O is

$$\frac{\left(x - \frac{x_e z}{f}\right)^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{f^2}. \tag{15}$$

At the distance d we have $x_{a'} = \frac{x_a d}{f}$, where $x_{a'}$ is the backprojection of x_a in the space. The projection of this ellipse in the space belongs to this surface (15) and to the plane $x = \frac{x_a d}{f}$. Their intersection gives us the equation of an ellipse:

$$\begin{cases} \frac{(z - z_c)^2}{a'^2} + \frac{y^2}{b'^2} = 1 \\ x = \frac{x_a d}{f}, \end{cases} \tag{16}$$

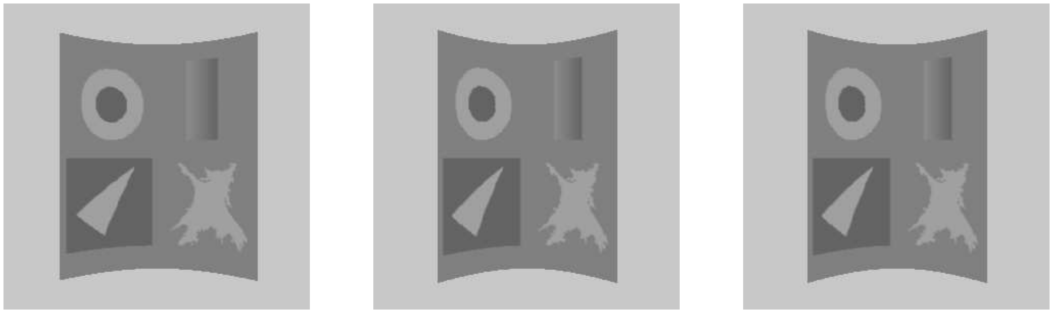

Fig. 5. (a)–(c) An image mapped on parabolic surfaces with different curvature.

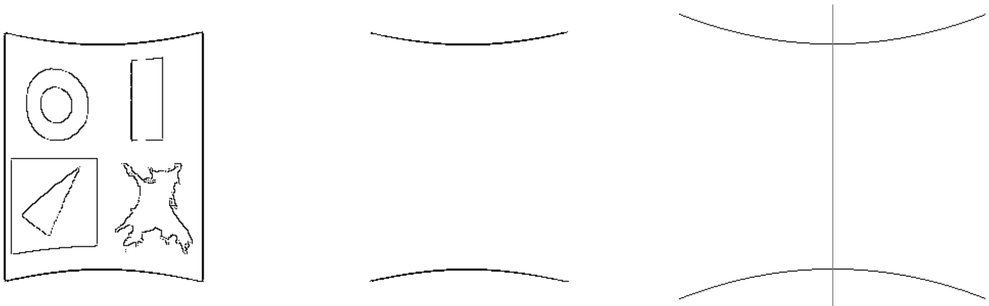

Fig. 6. (a) Edge detection, (b) Selection of two projected parallels, (c) Projection of the revolution axis.

where $a' = \frac{da}{x_e - a}$, $b' = \frac{bd}{f}\sqrt{\frac{x_e+a}{x_e-a}}$ and $z_c = \frac{dx_e}{x_e-a}$ are the parameters of this ellipse. Its center is $(\frac{x_a d}{f}, 0, \frac{dx_e}{x_e-a})$. If this equation is a circle, then we should have $a' = b'$:

$$f^2 = \frac{b^2(x_e^2 - a^2)}{a^2}. \tag{17}$$

We obtain a new relationship to find the focal distance f. Thus for each ellipse from the image, we obtain an ellipse in the space with a given scale factor. In the case of our surface having curvature different than zero in only one direction, we derive the equation of this surface from these curves by interpolation based on the least mean squares method. Next we can backproject the image on the surface in order to obtain the representation of the scene in the space.

§4. Simulation Results

In this section we illustrate the efficiency of the reconstruction method on two examples. The first example consists of a synthetic image mapped on different curved surfaces in order to validate our method. The second example consists of a painting on a vault in a church. For this purpose, we map the original image on three parabolic surfaces, as shown in Figures 5(a), (b), (c). After an edge detection shown in Figure 6(a), we select two parallel curves as shown Figure 6(b). The common normal is shown in Figure 6(c). In the Figures 7(a), (b) and (c), we compare the curvature of the real surface (dotted line) with the surface reconstructed with our method.

Figure 8(a) illustrates the result of an edge detection on the image shown Figure 1. The two axes are shown in the Figure 8(b). The reconstructed

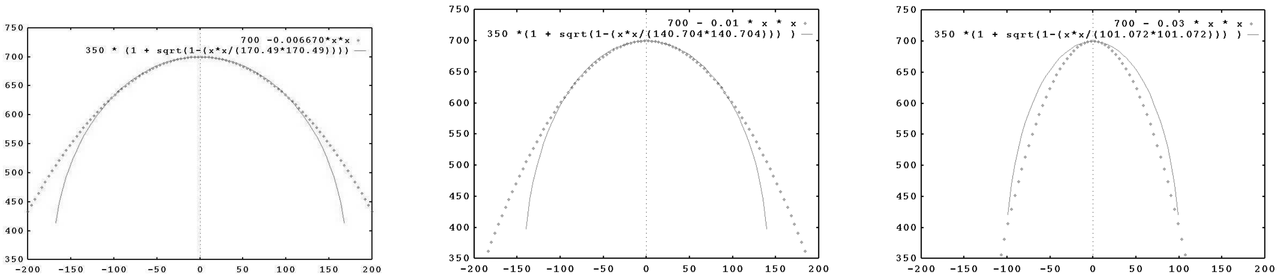

Fig. 7. (a), (b), (c) The reconstruction o :urves.

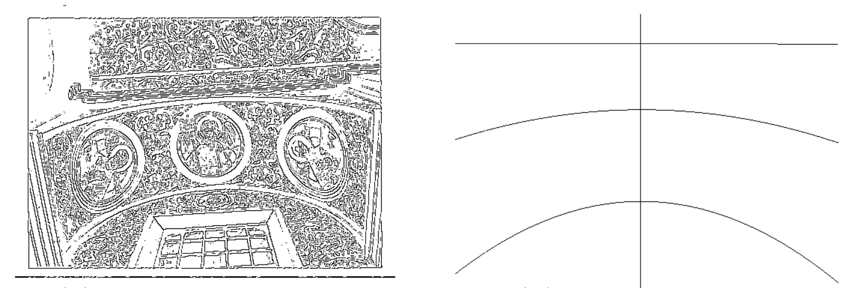

Fig. 8. (a) Edge detection of the Figure 1, (b) Curves and axes.

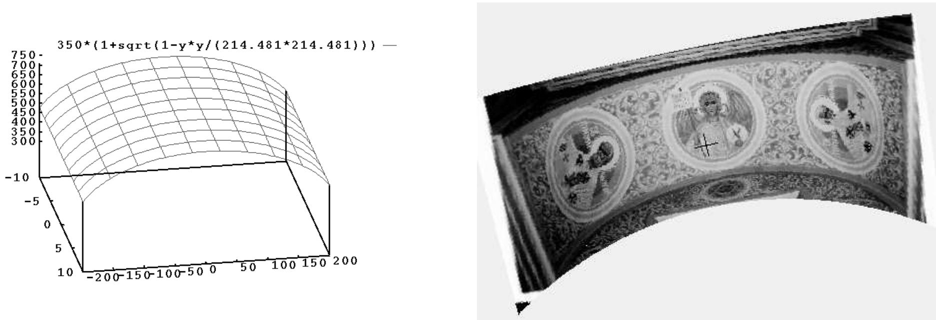

Fig. 9. (a) Reconstruction of the curved surface, (b) Synthetic visualization.

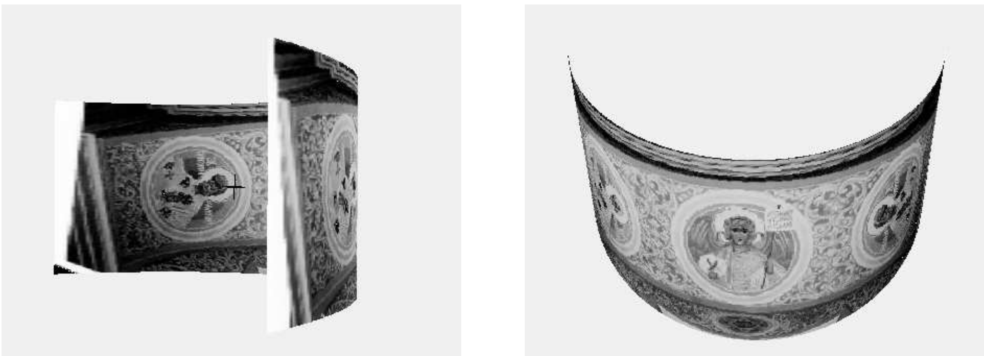

Fig. 10. (a), (b) Two other synthetic visualizations.

surface is shown in Figure 9(a). We are then able to obtain different views of this painting as depicted in Figures 9(a), 10(a) and (b).

§5. Conclusion

In this paper we propose a new approach for curved surface reconstruction based on a single perspective view. In the first step, we need an a priori knowledge about the projection in the image of the surface characterized by only one revolution axis for finding the localization of the curved surface. By using two projected parallels, we are able to locate the curved surface and then to reconstruct the shape in the space. We have shown how to find the curvature of the surface based on detecting parabolas and ellipses. This work is applied for the visualization of painting on columns or vaults. In this case we simulate a camera to obtain new images from different viewpoints. This method will be applied for artwork restoration [2] or mosaicing [1] for a mural painting on a vault or a scene around a column.

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