DISCRETE MULTICHANNEL ORTHOGONAL TRANSFORMS

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ABSTRACT

This paper presents a novel approach to the Fourier analysis of multichannel time series. Orthogonal matrix functions are introduced and are used in the definition of multichannel Fourier series of continuous-time periodic multichannel functions. Orthogonal transforms are proposed for discrete-time multichannel signals as well. The discrete-time one-dimensional multichannel transforms proposed in this paper are related to two-dimensional single-channel transforms, notably to the Discrete Fourier Transform and to the Discrete Hartley Transform. Therefore, fast algorithms for their computation can be easily constructed. Simulations on the use of Discrete Multichannel Transforms on color image compression have also been performed.

1. ORTHOGONAL MATRIX FUNCTIONS

Fourier analysis and digital signal transforms play an important role in digital signal/image processing and analysis [1], [2], [3]. All these transforms are used in single-channel digital signal/image processing. However, in recent years an increasing interest in multichannel signal processing and analysis has been observed [6]-[7]. Our approach to multichannel signal/image processing is to operate directly on the vector-valued (multichannel) signals [7]. A natural question that arises in this approach is the definition of multichannel signal transforms that can be used efficiently in digital signal/image compression applications.

The main contribution of this paper is the definition and analysis of the properties of multichannel transforms. An infinite series of real matrix functions: $\mathbf{\Phi}_0(t), \mathbf{\Phi}_1(t), \ldots, \mathbf{\Phi}_n(t), \ldots$ having dimensions $p \times p$ is said to be *orthogonal* on the interval $[a, b]$ if:

$$\int_a^b \mathbf{\Phi}_n(t)\mathbf{\Phi}_m(t)dt = \mathbf{0} \qquad n \neq m \tag{1}$$

The system is said to be orthonormal if:

$$\int_a^b \mathbf{\Phi}_n^2(t) = \mathbf{I} \tag{2}$$

for $n = 0, 1, 2, \ldots$. Let $\mathbf{x}(t) = [x_1(t), \ldots, x_p(t)]^T$ be a multichannel function (time series) defined in the interval $[a, b]$. It can be represented as a sum of a series of the orthonormal matrix functions:

$$\mathbf{x}(t) = \mathbf{\Phi}_0(t)\mathbf{c}_0 + \mathbf{\Phi}_1(t)\mathbf{c}_1 + \ldots + \mathbf{\Phi}_n(t)\mathbf{c}_n + \ldots \tag{3}$$

The Fourier vector coefficients $\mathbf{c}_i, \ i = 0, 1, \ldots, n, \ldots$ as given by:

$$\mathbf{c}_n = \int_a^b \mathbf{\Phi}_n(t)\mathbf{x}(t)dt \tag{4}$$

The vectors $\mathbf{c}_n, \quad n = 0, 1, 2, \ldots,$ are called Fourier vector coefficients of $\mathbf{x}(t)$ and the corresponding series is called Fourier series of the multichannel function $\mathbf{x}(t)$. We have proven that convergence and completeness theorems of the series (3) hold and are similar to the corresponding theorems of the classical Fourier series.

The orthogonal matrix functions and the related Fourier series that have been described in this section are very general. In the following section we shall describe a specific form of multichannel Fourier series based on orthogonal trigonometric matrix functions. The basic trigonometric matrix functions to be used in this paper are the following:

$$\mathbf{\Phi}_0(t) = \frac{1}{\sqrt{p}}(\mathbf{C} + \mathbf{S}) = \frac{1}{\sqrt{p}} \cdot \tag{5}$$

$$\begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & cos(\frac{2\pi}{p}) + sin(\frac{2\pi}{p}) & \ldots & cas(\frac{2\pi(p-1)}{p}) \\ 1 & cos(\frac{4\pi}{p}) + sin(\frac{4\pi}{p}) & \ldots & cas(\frac{4\pi(p-1)}{p}) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & cas(\frac{2\pi(p-1)}{p}) & \ldots & cas(\frac{2\pi(p-1)^2}{p}) \end{bmatrix}$$

where $cas(x) = \cos(x) + \sin(x)$.

$$\mathbf{\Phi}_n(t) = \frac{1}{\sqrt{p}}(\mathbf{C}cosnt + \mathbf{S}sinnt) = \frac{1}{\sqrt{p}} \cdot \tag{6}$$

$$\begin{bmatrix} cosnt & cosnt & \ldots & cosnt \\ cosnt & cos(nt - 2\pi/p) & \ldots & cos(nt - 2\pi(p-1)/p)) \\ cosnt & cos(nt - 4\pi/p) & \ldots & cos(nt - 4\pi(p-1)/p)) \\ \vdots & \vdots & \vdots & \vdots \\ cosnt & cos(nt - 2\pi(p-1)/p) & \ldots & cos(nt - 2\pi(p-1)^2/p) \end{bmatrix}$$

$$\phi_n(t) = \frac{1}{\sqrt{p}}(\mathbf{C}sinnt - \mathbf{S}cosnt) = \frac{1}{\sqrt{p}} \cdot \tag{7}$$

$$\begin{bmatrix} sinnt & sinnt & \ldots & sinnt \\ sinnt & sin(nt - 2\pi/p) & \ldots & sin(nt - 2\pi(p-1)/p)) \\ sinnt & sin(nt - 4\pi/p) & \ldots & sin(nt - 4\pi(p-1)/p)) \\ \vdots & \vdots & \vdots & \vdots \\ sinnt & sin(nt - 2\pi(p-1)/p) & \ldots & sin(nt - 2\pi(p-1)^2/p) \end{bmatrix}$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & cos(2\pi/p) & \ldots & cos(2\pi(p-1)/p)) \\ 1 & cos(4\pi/p) & \ldots & cos(4\pi(p-1)/p)) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & cos(2\pi(p-1)/p) & \ldots & cos(2\pi(p-1)^2/p) \end{bmatrix} \tag{8}$$

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$$\mathbf{S}=\begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & sin(2\pi/p) & \dots & sin(2\pi(p-1)/p)) \\ 0 & sin(4\pi/p) & \dots & sin(4\pi(p-1)/p)) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & sin(2\pi(p-1)/p) & \dots & sin(2\pi(p-1)^2/p) \end{bmatrix} \quad (9)$$

The orthogonal system consists of the functions $\mathbf{\Phi}_0, \mathbf{\Phi}_1, \dots, \mathbf{\Phi}_n, \dots, \phi_1, \dots, \phi_n, \dots$. Orthogonality can be easily proven. In the case of the three channel signals ($p=3$), which is of special interest in color image processing, the basic trigonometric matrix functions take the form:

$$\mathbf{\Phi}_0(t)=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & (-1+\sqrt{3})/2 & (-1-\sqrt{3})/2 \\ 1 & (-1-\sqrt{3})/2 & (-1+\sqrt{3})/2 \end{bmatrix} \quad (10)$$

$$\mathbf{\Phi}_n(t)=\frac{1}{\sqrt{3}}\begin{bmatrix} cosnt & cosnt & cosnt \\ cosnt & cos(nt-2\pi/3) & cos(nt-4\pi/3) \\ cosnt & cos(nt-4\pi/3) & cos(nt-2\pi/3) \end{bmatrix} \quad (11)$$

$$\phi_n(t)=\frac{1}{\sqrt{3}}\begin{bmatrix} sinnt & sinnt & sinnt \\ sinnt & sin(nt-2\pi/3) & sin(nt-4\pi/3)) \\ sinnt & sin(nt-4\pi/3) & sin(nt-2\pi/3)) \end{bmatrix} \quad (12)$$

Let $\mathbf{x}(t)$ be a p-channel periodic function of period 2π. Its multichannel trigonometric Fourier series expansion is given by:

$$\mathbf{x}(t)=\frac{1}{2}\mathbf{\Phi}_0(t)\mathbf{a}_0+\sum_{n=1}^{\infty}(\mathbf{\Phi}_n(t)\mathbf{a}_n+\phi_n(t)\mathbf{b}_n) \quad (13)$$

It can be easily proven that the Fourier vector coefficients $\mathbf{a}_0, \mathbf{a}_1, \dots, \mathbf{a}_n, \dots, \mathbf{b}_1, \dots, \mathbf{b}_n, \dots$ are given by:

$$\mathbf{a}_0=\frac{1}{\pi}\int_{-\pi}^{\pi}\mathbf{\Phi}_0(t)\mathbf{x}(t)dt \quad (14)$$

$$\mathbf{a}_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\mathbf{\Phi}_n(t)\mathbf{x}(t)dt \quad (15)$$

$$\mathbf{b}_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\phi_n(t)\mathbf{x}(t)dt \quad (16)$$

The vector Fourier coefficients are proven to be linearly related to the classical single-channel Fourier coefficients.

The transformation matrix $\mathbf{\Phi}_0$ given by (5) is essentially the Discrete Hartley Transform (DHT) [5]. The DHT:

$$\mathbf{v}=\mathbf{\Phi}_0\mathbf{u} \quad (17)$$

is unitary and diagonalizes the covariance matrix $\mathbf{R}_u$ of the form:

$$\mathbf{R}_u=\begin{bmatrix} 1 & \rho & \rho & \dots & \rho \\ \rho & 1 & \rho & \dots & \rho \\ \rho & \rho & 1 & \dots & \rho \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho & \rho & \rho & \dots & 1 \end{bmatrix} \quad (18)$$

for $0<\rho<1$, i.e. when there is the same covariance between the various vector elements. Thus, the DHT $\mathbf{\Phi}_0$ is the Karhunen-Loeve transformation of a random vector having covariance matrix of the form (18). Most of the energy is concentrated in $\sigma_v^2(0)$. The unitary transform for three channel signals has the form:

$$\mathbf{\Phi}_0=\begin{bmatrix} 0.5773 & 0.5773 & 0.5773 \\ 0.5773 & 0.2113 & -0.788 \\ 0.5773 & -0.788 & 0.2113 \end{bmatrix} \quad (19)$$

A similar transform has been presented by Pratt et al. [4] for color RGB image compression:

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}=\begin{bmatrix} 0.575 & 0.615 & 0.540 \\ 0.608 & 0.120 & -0.785 \\ 0.548 & -0.779 & 0.305 \end{bmatrix}\begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (20)$$

The resulting transformation is called K1K2K3 transformation. The first component of the transformation is clearly related to the image luminance. The unitary transformation of 4-channel vector takes the form:

$$\mathbf{\Phi}_0=\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (21)$$

It is directly related to the Hadamard matrix of order 4 [1], [2]:

$$\mathbf{H}_2=\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

2. DISCRETE MULTICHANNEL TRANSFORMS (DMT)

In most cases in digital signal and image processing applications we deal with discrete multichannel signals of the form $\mathbf{x}=[\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{N-1}]^T$. Therefore, it is natural to define discrete multichannel transforms of these signals of the form:

$$\mathbf{X}=\mathbf{\Phi}\mathbf{x} \quad (22)$$

where $\mathbf{\Phi}$ is a $pN \times pN$ transformation matrix. The transform is unitary if

$$\mathbf{\Phi}^T\mathbf{\Phi}=\mathbf{I} \quad (23)$$

Let us denote by $\mathbf{X}_k, k=0,\dots,N-1$ the transform vector coefficients:

$$\mathbf{X}_k=\sum_{n=0}^{N-1}\mathbf{\Phi}_{kn}\mathbf{x}_n \quad (24)$$

The inverse transform is given by:

$$\mathbf{x}_n=\sum_{k=0}^{N-1}\mathbf{\Phi}_{kn}\mathbf{X}_k \quad (25)$$

A family of discrete multichannel transforms of the form (22) can be obtained by using the following form of transform matrix $\mathbf{\Phi}=\{\Phi_{nk}\}$, where:

$$\mathbf{\Phi}_{nk}=\frac{1}{\sqrt{pN}}\cdot \quad (26)$$

$$\begin{bmatrix} cas\frac{2(a+bn)k\pi}{cN+d} & \dots & cas\frac{2(a+bn)k\pi}{cN+d} \\ cas\frac{2(a+bn)k\pi}{cN+d} & \dots & cas\left[\frac{2(a+bn)k\pi}{cN+d}-\frac{2\pi(p-1)}{p}\right] \\ \vdots & & \vdots \\ cas\frac{2(a+bn)k\pi}{cN+d} & \dots & cas\left[\frac{2(a+bn)k\pi}{cN+d}-\frac{2\pi(p-1)^2}{p}\right] \end{bmatrix}$$

where a, b, c, d are arbitrary integer parameters. By proper choice of the transform parameters, we can construct an entire family of *Discrete Multichannel Transforms*. It can easily be proven that the submatrices $\mathbf{\Phi}_{nk}$ for $d=0$ can be expressed as:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}\left[\mathbf{C}\left(cos\frac{2(a+bn)k\pi}{cN} + sin\frac{2(a+bn)k\pi}{cN}\right) + \mathbf{S}\left(sin\frac{2(a+bn)k\pi}{cN} - cos\frac{2(a+bn)k\pi}{cN}\right)\right] \quad (27)$$

The submatrix $\mathbf{\Phi}_{n0}$ has the form:

$$\mathbf{\Phi}_{n0} = \frac{1}{\sqrt{pN}}\left(\mathbf{C}-\mathbf{S}\right) \quad (28)$$

The parameters a, b, c, d must be chosen in such a way so that the transform is unitary. It was proven that sufficient conditions for this purpose are the following:

$$b/c = 1, \qquad d = 0, \qquad 2a/c \in \mathcal{Z} \quad (29)$$

where $\mathcal{Z}$ is the set of integer numbers. According to these conditions, the cas functions in (26) take the form:

$$cas\frac{2(a+bn)k\pi}{bN} = cas\frac{(2n+z)k\pi}{N} \quad (30)$$

where $z = 2a/b \in \mathcal{Z}$. The submatrices of the transform become:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}\cdot \quad (31)$$

$$\begin{bmatrix} cas\frac{(2n+z)k\pi}{N} & \dots & cas\frac{(2n+z)k\pi}{N} \\ cas\frac{(2n+z)k\pi}{N} & \dots & cas\left[\frac{(2n+z)k\pi}{N} - \frac{2\pi(p-1)}{p}\right] \\ \vdots & & \vdots \\ cas\frac{(2n+z)k\pi}{N} & \dots & cas\left[\frac{(2n+z)k\pi}{N} - \frac{2\pi(p-1)^2}{p}\right] \end{bmatrix}$$

A large number of multichannel transforms can be devised by appropriate choice of the transform parameter z. The transform

$$\mathbf{\Phi}_{nk} = \mathbf{\Phi}_{kn} = \frac{1}{\sqrt{pN}}\cdot \quad (32)$$

$$\begin{bmatrix} cas(\frac{2\pi kn}{N}) & \dots & cas(\frac{2\pi kn}{N}) \\ cas(\frac{2\pi kn}{N}) & \dots & cas(\frac{2\pi kn}{N} - \frac{2\pi(p-1)}{p}) \\ \vdots & \vdots & \vdots \\ cas(\frac{2\pi kn}{N}) & \dots & cas(\frac{2\pi kn}{N} - \frac{2\pi(p-1)^2}{p}) \end{bmatrix}$$

$$= \frac{1}{\sqrt{pN}}[\mathbf{C}cas(\frac{2\pi kn}{N}) + \mathbf{S}(sin(\frac{2\pi kn}{N}) - cos(\frac{2\pi kn}{N})]$$

is essentially a special case of (26) for $z=0$. Therefore, it will be denoted Discrete Multichannel Transform 0 (DMT0). A symmetry property of the transform submatrices is given by:

$$\mathbf{\Phi}_{n+N/2,k} = (-1)^k\mathbf{\Phi}_{nk} \quad (33)$$

$$\mathbf{\Phi}_{n,k+N/2} = (-1)^n\mathbf{\Phi}_{nk} \quad (34)$$

The transformation matrix $\mathbf{\Phi}_{nk}$ can be expressed as follows:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}\Big[Re\{\mathbf{W}_p\}(\frac{1}{2}(w_N^{*nk} + w_N^{nk}) + \frac{i}{2}(w_N^{nk} - w_N^{*nk})) - Im\{\mathbf{W}_p\}(\frac{i}{2}(w_N^{nk} - w_N^{*nk}) - \frac{1}{2}(w_N^{nk} + w_N^{*nk}))\Big] \quad (35)$$

where $Re\{\mathbf{W}_p\}, Im\{\mathbf{W}_p\}$ denote the real and the imaginary parts of the DFT matrix $\mathbf{W}_p$.

The transform submatrices $\mathbf{\Phi}_{nk}$ for $z=1$ take the form:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}\cdot \quad (36)$$

$$\begin{bmatrix} cas\frac{(2n+1)k\pi}{N} & \dots & cas\frac{(2n+1)k\pi}{N} \\ cas\frac{(2n+1)k\pi}{N} & \dots & cas\left[\frac{(2n+1)k\pi}{N} - \frac{2\pi(p-1)}{p}\right] \\ \vdots & & \vdots \\ cas\frac{(2n+1)k\pi}{N} & \dots & cas\left[\frac{2(n+1)k\pi}{N} - \frac{2\pi(p-1)^2}{p}\right] \end{bmatrix}$$

This transform will be denoted subsequently as Discrete Multichannel Transform 1 (DMT1). The terms of these submatrices that correspond to the temporal axis n have forms $\cos[(2n+1)k\pi/N]$, $\sin[(2n+1)k\pi/N]$. The first of them is similar to the kernel function of the Discrete Cosine Transform. The multichannel transform DMT1 is directly related to the Discrete Fourier Transforms of sizes p, N as follows:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}[Re(\mathbf{W}_p)\{\frac{1}{2}(w_N^{*(n+\frac{1}{2})k} + w_N^{(n+\frac{1}{2})k}) + \frac{i}{2}(w_N^{(n+\frac{1}{2})k} - w_N^{*(n+\frac{1}{2})k)}\} - Im(\mathbf{W}_p)\{\frac{i}{2}(w_N^{(n+\frac{1}{2})k} - w_N^{*(n+\frac{1}{2})k}) - \frac{1}{2}(w_N^{(n+\frac{1}{2})k} + w_N^{*(n+\frac{1}{2})k})\}] \quad (37)$$

where $\mathbf{W}_p$ is the transform matrix of DFT, with length p and $w_N = exp\left(-j\frac{2\pi}{N}\right)$. Finally, the submatrices of DMT1 possess the following symmetry:

$$\mathbf{\Phi}_{n+\frac{N}{2},k} = (-1)^k\mathbf{\Phi}_{nk} \quad (38)$$

Thus, the calculation of the half of the matrix $\mathbf{\Phi}$ is sufficient for its determination.

For $z=2$ the transform submatrices $\mathbf{\Phi}_{nk}$ take the form:

$$\mathbf{\Phi}_{nk} = \frac{1}{\sqrt{pN}}\cdot \quad (39)$$

$$\begin{bmatrix} cas\frac{2(n+1)k\pi}{N} & \dots & cas\frac{2(n+1)k\pi}{N} \\ cas\frac{2(n+1)k\pi}{N} & \dots & cas\left[\frac{2(n+1)k\pi}{N} - \frac{2\pi(p-1)}{p}\right] \\ \vdots & & \vdots \\ cas\frac{2(n+1)k\pi}{N} & \dots & cas\left[\frac{2(n+1)k\pi}{N} - \frac{2\pi(p-1)^2}{p}\right] \end{bmatrix}$$

This transform will be denoted subsequently as Discrete Multichannel Transform 2 (DMT2). DMTs for other parameter values can be defined similarly.

3. SIMULATIONS

One of the aims of this paper is to define multichannel orthogonal transforms that could eventually be used in digital multichannel signal/image compression. One of the primary applications of this work could be on color (RGB) image compression. Therefore, we have performed extensive simulations on the use of these transforms in this area. We have experimented both with one-dimensional and two-dimensional signals by using DMT1 and DMT2 transforms. For one-dimensional transforms we have used definition (24). We have used two-dimensional transforms of size $N \times M$ having the form:

$$\mathbf{X}_{kl} = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \mathbf{\Phi}_{N,kn} \mathbf{\Phi}_{M,lm} \mathbf{x}_{nm} \tag{40}$$

where $\mathbf{\Phi}_{N,kn}$, $\mathbf{\Phi}_{M,lm}$ denote kernels for transforms of size N, M respectively. $\mathbf{x}_{nm}$, $\mathbf{X}_{kl}$ denote color image pixels and transform coefficients respectively. We have used transform sizes 8×8, 16×16, 32×32 that are usually employed in image transform coding. We have employed the Mean Square Variance (MSV) to study energy concentration in the transform domain, defined as:

$$\sigma_{kl}^2 = E(\mathbf{X}_{kl}^T \mathbf{X}_{kl}) \tag{41}$$

where $E(\cdot)$ denotes the expectation operator. MSV was estimated from the transform coefficients $\mathbf{X}_{kl}$ averaged for all the blocks of an image. It has been observed that, unlikely to the DCT, the DMT coefficients having maximal MSVs tend to be concentrated both in low and in high frequencies in the one-dimensional case. In the two-dimensional case they are concentrated in the four corners of the transform domain. This property is similar to the energy concentration in the DFT domain. The DMT1 transform shows very good energy compaction. However, as already stated, the energy is concentrated in the four corners of the transform domain. On the contrary, DCT concentrates the energy in only one corner of the DCT domain (low frequency corner). Thus, transform vectors coming from all four corners of the DMT domain must be retained during image compression. One of our primary aims was the comparison of our method with DCT-based transform coding. The DMT transforms are applied directly to the RGB color image. Both DMT1 and DMT2 were employed. The DCT transform has been applied to the color image channels after a YC_BC_R transform or a YIQ transform or the $K1K2K3$ transform that are frequently employed in digital video applications [2]. These transforms perform channel decorrelation, at least partially. Both in the DMT and in the DCT we retain the transformed coefficient vectors that possess the largest energy. As said, in the case of DMT, such vectors are close to any of the corners of the transform domain. No quantization is performed on the retained coefficients, since our aim was to study only the energy compaction capabilities of the proposed transform. The rest of the coefficients are zeroed and discarded. The color image is reconstructed by the inverse procedure. Let us denote by $MSE = \| \hat{\mathbf{x}} - \mathbf{x} \|$ the mean square error between the original image and the reconstructed image $\hat{\mathbf{x}}$. The figure of merit that is used for comparison is the PSNR (in dB):

$$\text{PSNR (dB)} = 10 \log \left\{ \frac{255^2}{MSE} \right\} \tag{42}$$

versus the percentage of the transform coefficient vectors that are retained. Several test images have been used for this study. Both test color images and synthetic color images have been used to assess compression performance. Generally speaking, it has been found that the multichannel transform DMT2 had similar performance to that of the DCT combined with a channel decorrelation transform (e.g. YC_BC_R). In certain cases DMT2 outperformed DCT. However, this is not a general rule. Furthermore, it has been found that DMT2 consistently outperformed DMT1 and DMT0. It must be noted that the family of multichannel transforms defined in this paper is very rich. The work reported in this paper did not make exhaustive comparisons for all the multichannel transforms of this family. Thus, it is still an open question if some of these transforms outperform DCT-based compression methods.

8. CONCLUSIONS

A system of orthogonal matrix functions has been defined in this paper. This system can be used to obtain the Fourier series analysis of multichannel continuous time signals. The properties of the multichannel Fourier series are presented. It is shown that such multichannel series have similar properties to the classical Fourier series. Trigonometric matrix functions are defined and used to obtain trigonometric multichannel Fourier series. It is proven that these series are related to the classical single-channel trigonometric Fourier series. Furthermore, the trigonometric matrix functions are related to unitary single-channel transforms. The sampling of multichannel matrix functions leads to the definition of multichannel discrete-time transforms. Multichannel non-separable transforms are defined and their orthogonality is proven. The one-dimensional multichannel transform is related to two-dimensional single-channel transforms, notably to the 2-d Discrete Fourier Transform and to the 2-d Discrete Hartley Transform. This relation can be used to construct fast transform algorithms. A family of Discrete Multichannel transforms (DMTs) has also been presented that can be used in multichannel image compression. The analysis presented in this paper provides a very general framework for the definition of other multichannel transforms as well. The derivation of such transforms is the subject of ongoing research. The motivation of this on-going research is to define multichannel non-separable transforms that can be used efficiently in multichannel signal/image compression.

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