EDGE DETECTION OPERATORS FOR ANGULAR DATA

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ABSTRACT

Physical quantities referring to angles, like vector direction, color hue etc., are periodic in nature. Due to this periodicity edge detectors proposed for data on the line cannot be used to detect edges in angle-related signals. In this paper we use estimators of circular dispersion to introduce edge detectors for angular signals and discuss their application in edge detection on hue images. Extensions of the notion of quasi-range to circular data are also proposed. These "circular" quasi-ranges have good and user-controlled properties as edge detectors on noisy angular signals. The performance of the proposed edge operators is evaluated on angular edges, using both qualitative and quantitative criteria.

1. INTRODUCTION

The great majority of the multichannel signal processing and analysis literature deals with signals represented in a cartesian coordinate system. However, vector representation by means of vector direction angles and vector magnitude, i.e. representation in a polar coordinate system, can provide a more natural context to deal with certain problems. Applications that involve vector direction manipulation (e.g. estimation of motion direction from noisy motion vector fields) can be good candidates for treatment in polar coordinates. Another important application comes from the area of color image processing: in HSI and HSV color representation systems, that are widely used in graphic arts, colorimetry etc., hue is essentially a measure of direction. Recently, filters for vector-valued signals that filter the vector direction angle and the vector magnitude separately have been proposed [1]-[3]. Vector direction, as well as other physical quantities referring to angles, exhibit an inherently periodic nature and can be regarded as observations on a circle having unit radius. Due to this periodicity, the statistical theory used for data on the line cannot be used to deal with such quantities. For this kind of data, called *angular*, *directional* or *circular* data, a special statistical theory, the theory of angular statistics, has been developed. A complete review of angular statistics can be found in [4].

2-D angular signals will be called angular images. The hue component of a color image in HSI, HSV color coordinate systems can be considered to be an angular image. Images depicting the direction angle of vector fields are angular as well. Analysis of such images can be of particular interest in many applications. Detection of edges on the hue color component can be useful in cases like color object recognition, color image segmentation etc. Segmentation of angular images depicting the motion direction of motion vector fields can be used for direction-based vehicle identification and tracking. In the following we shall concentrate on edge detection on angular images and discuss its application in hue image edge detection. Detection of edges on angular images cannot be done using the edge detectors proposed for grayscale images (e.g. Sobel edge detectors). Such detectors, when applied on a hue image, would erroneously detect an edge of height 350^0 between two homogeneous hue regions having values 5^0 and 355^0 respectively, while the real edge height would be 10^0 i.e. the smallest arc joining the points related to angles 5^0, 355^0 on the unit circle. Therefore, new edge detectors have to be introduced for data of periodic nature. In this paper we use estimators of circular dispersion to introduce edge detectors for angular data. Furthermore we propose two ways to extend the notion of quasi-range to circular data and study their theoretical properties as well as their performance as edge detectors.

2. MEASURES OF ANGULAR DISPERSION

Circular measures of dispersion can be used as edge detectors for angular data, since edges are areas having large local signal variance. The *circular mean difference* $\bar{D}_0$ [4] of N angular observations $X_1, \ldots, X_N$ $(0 \leq X_i < 2\pi)$ is such a measure. It is defined by the following relation :

$$\bar{D}_0 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_{ij} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\{\pi - |\pi - |X_i - X_j||\} \tag{1}$$

where ξ_{ij} is the deviation between two angles i.e. the smallest of the two angles that are defined by the points X_i and X_j on the unit circle. Another dispersion measure described in [4] is the *circular range*. It is defined as the length of the smallest arc on the unit circle which includes all the angular observations. The circular range can be evaluated in the following way : We order the directional samples $X_1, X_2, \cdots, X_N$ and define the measures of the arcs T_i between adjacent samples as follows :

$$\begin{aligned} T_i &= X_{(i+1)} - X_{(i)},\ i = 1, \cdots, N-1, \\ T_N &= 2\pi - X_{(N)} + X_{(1)} \end{aligned} \tag{2}$$

where $X_{(i)}$ denotes the i-th ordered sample. The circular range is then given by:

$$w = 2\pi - \max(T_1, \dots, T_N). \tag{3}$$

The above-mentioned measures of dispersion can be used to detect angular edges, since their output value will be big in edge areas and small in homogeneous regions. Unfortunately, these edge detectors have poor performance when acting on noisy angular images. In the following, we shall introduce extensions of the circular range, i.e. circular quasi-ranges, that are suitable for edge detection on noisy signals. Their big advantage is that they allow noise suppression to be done along with edge detection, thus making the edge detector more robust to noise. Noise suppression is achieved by leaving out a certain amount of extreme points (that depends on the order of the quasi-range).

In the case of data on the line, the i-th quasi-range of N observations is defined by the following relation :

$$Z_i = X_{(N-i)} - X_{(i+1)}. \tag{4}$$

The first way to extend this definition to circular data is by making use of the circular range (3). The circular quasi-range w_i of order i, $(0 \leq i < (N-1)/2,\ N$ odd) can be found by evaluating the circular range w and "stripping" out i data points from each of the two ends of the arc w. The arc that includes all data points except for the $2 \cdot i$ points that have been rejected, will be called the i-th order *circular quasi-range* (CQR).

The evaluation of the output pdf for the circular quasi-range, when this is applied on a noisy angular edge modeled as a step function, is very cumbersome for a general noise distribution. We have evaluated the theoretical output pdf of a CQR operator of length N and order i for the tractable case of impulsive noise of constant impulse value that corrupts a one-dimensional angular edge modeled as a step function. Even in this simplified case, the expression for the above-mentioned pdf is very lengthy. The theoretical and experimental mean for a circular quasi-range of order $i = 1$ and window length $N = 5$ applied on a one-dimensional angular edge of height 60^0 corrupted by impulsive noise having probability of occurrence $p = 10\%$ is presented in Figure 1. It can be seen that there is perfect correspondence between theoretical and simulation results. The large mean output value of the CQR response on the edge, compared to the small mean output value in the homogeneous regions, clearly indicates that CQR can be used as an edge detector.

Another definition for the circular quasi-range of N angular data points can be the following : We define as *modified circular quasi-range* (MCQR) W_i of order i the smallest arc that contains all points except for $2 \cdot i$ successive points. For $i = 0$, MCQR coincides with the circular range (3). Modified circular quasi-range W_i can be evaluated in the following way: We order the directional samples $X_1, X_2, \cdots, X_N$ and evaluate arc lengths S_i between points being $\Delta = 2i + 1$ samples apart in a circular manner i.e. :

$$S_k = \begin{cases} X_{(\Delta+k)} - X_{(k)} & 1 \leq k \leq N - \Delta \\ 2\pi + X_{(mod_N(\Delta+k))} - X_{(k)} & N - \Delta < k \leq N \end{cases} \tag{5}$$

W_i is then given by :

$$W_i = 2\pi - S_{max} = 2\pi - max(S_1, \dots, S_N). \tag{6}$$

We have proven that the output pdf f_{W_0} of the one-dimensional MCQR W_0 (i.e the circular range) for constant signal areas corrupted by noise is given by :

$$\begin{gathered} f_{W_0}(x) = f_{T_{max}}(2\pi - x), \\ f_{T_{max}}(x) = \\ N! \cdot \Big(\int_0^x \dots \int_0^x \{ \int_0^{2\pi} f(v - v_{N-1} - \dots - v_2 - x) \cdot \\ f(v - v_{N-1} - \dots - v_2) \dots f(v - v_{N-1}) f(v) dv \} \cdot \\ u(2\pi - x - v_2 - \dots - v_{N-1}) \cdot \\ u(2x - 2\pi + v_2 + \dots + v_{N-1}) dv_2 \dots dv_{N-1} + \\ \int_0^x \dots \int_0^x \{ \int_0^{2\pi} f(v - v_{N-1} - \dots - x - v_1) \cdot \\ f(v - v_{N-1} - \dots - x) \dots f(v - v_{N-1}) f(v) dv \} \cdot \\ u(2\pi - v_1 - x - v_3 - \dots - v_{N-1}) \cdot \\ u(2x - 2\pi + v_1 + v_3 + \dots + v_{N-1}) dv_1 dv_3 \dots dv_{N-1} + \\ \dots + \int_0^x \dots \int_0^x \{ \int_0^{2\pi} f(v - x - v_{N-2} - \dots - v_1) \cdot \\ f(v - x - v_{N-2} - \dots - v_2) \dots f(v - x) f(v) dv \} \cdot \\ u(2\pi - v_1 - v_2 - \dots - v_{N-2} - x) \cdot \\ u(2x - 2\pi + v_1 + \dots + v_{N-2}) dv_1 \dots dv_{N-2} + \\ \int_0^x \dots \int_0^x \{ \int_0^{2\pi} f(v - 2\pi + x) \cdot \\ f(v - v_{N-1} - \dots - v_2) \dots f(v - v_{N-1}) f(v) dv \} \cdot \\ u(2\pi - v_2 - \dots - v_{N-1} - x) \cdot \\ u(2x - 2\pi + v_2 + \dots + v_{N-1}) dv_2 \dots dv_{N-1} \Big) \end{gathered} \tag{7}$$

where $f(x)$ is the input data distribution and $u(x)$ the unit step function. The pdf of W_i for $i > 0$ can also be derived using the same methodology but results will be omitted because the corresponding formulae are rather lengthy. The theoretical output pdf of the circular range for length $N = 3$ when the noise is distributed according to von Mises distribution [4] can be seen in Figure 2 along with the experimentally evaluated pdf. Using the derived pdf we can evaluate the mean and variance of the edge detector output in constant signal areas and, consequently, choose an appropriate threshold so that the output of the detector at homogeneous regions does not exceed this threshold. Evaluation of the output pdf for the actual edge region would be of equal importance. However, this is difficult to be done, since it requires order statistics from dependent, non-identically distributed random variables.

3. EXPERIMENTAL PERFORMANCE EVALUATION

The proposed edge detectors have been applied on the hue component of noisy color images. Figure 4 shows the output of a 5×5 MCQR edge detector of order $i = 4$ applied on the hue component of test image "Pepper" that is corrupted by 10% impulsive noise (Fig. 3). The MCQR output has been thresholded using $T = 16$. It can be seen that the performance of the MCQR edge detector in the presence of noise is very good. Quantitative performance evaluation of the proposed edge detectors in the presence of noise has also been pursued. Usually, the output of an edge detector is thresholded and pixels with value over the threshold are considered as edge points. The thresholding operation is therefore crucial for the performance of the edge operator and the choice of the optimum threshold T_{opt} for a certain type of noise must be done carefully. For a given threshold T the performance of an edge detector can be described in terms of three probabilities i.e. the probability of false alarm, the probability of detection and the probability of non-detection. The probability of false alarm $P_F(T)$ is the probability of a non-edge point to be classified as an edge. The probability of detection $P_D(T)$ is the probability that an edge point is detected as such. Finally the probability of non-detection $P_{ND}(T)$ is the probability an edge point not to be detected as such, i.e., to be classified in the background. In order to assess the performance of the proposed edge detectors we have used the following ratio :

$$R(T) = \frac{P_F(T) + P_{ND}(T)}{P_D(T)}. \qquad (8)$$

The smaller the value of $R(T)$ the better the performance of an edge detector. The optimum threshold T_{opt} is the one that minimizes $R(T)$. The performance of different detectors on the same noise type can be assessed by comparing the minimum value $R_{min} = R(T_{opt})$. In our experiments, we used R_{min} to evaluate the performance of CQR and MCQR edge detectors of various orders on synthetic hue edges. Comparisons were limited only to these two detectors because, as it was mentioned earlier, edge operators for data on the line are not appropriate for angular data. A two-dimensional RGB signal whose cross-section contains a step edge was used in the simulations :

$$\begin{aligned} Red(k) &= \begin{cases} 80 & 0 \le k < M/2 \\ 50 & M/2 \le k \le M \end{cases} \\ Green(k) &= \begin{cases} 50 & 0 \le k < M/2 \\ 80 & M/2 \le k \le M \end{cases} \qquad (9) \\ Blue(k) &= 20. \end{aligned}$$

If we transform this signal to HSV we obtain a hue edge whose cross-section is :

$$H(k) = \begin{cases} 30^0 & 0 \le k < M/2 \\ 90^0 & M/2 \le k \le M \end{cases} \qquad (10)$$

Impulsive noise having uniformly distributed impulse value and probability of occurrence $p = 10\%$, $p = 20\%$ was used to contaminate independently each of the three channels of the RGB signal. After that we transformed the noisy signal to the HSV domain and applied the CQR and MCQR edge detectors of various orders i on the hue signal, using windows of size 3×3 and 5×5. Results are presented in Figures 5,6 where R_{min} is plotted as a function of order i. It can be seen that MCQR and CQR edge detectors have almost the same performance for this noise type. However, MCQR was found to be better than CQR for impulsive noise of constant impulse value. The plots indicate also that the order for which the best performance is achieved depends on the noise power. The bigger the power of the noise, the greater the order must be in order the edge detector to be able to reject the noisy points. Note that, for the same type of noise, MCQR achieves its minimum R_{min} value for smaller order i. Diagrams also show that the 5×5 edge detectors have better performance than the 3×3 counterparts and that the performance decreases with the noise power.

4. REFERENCES

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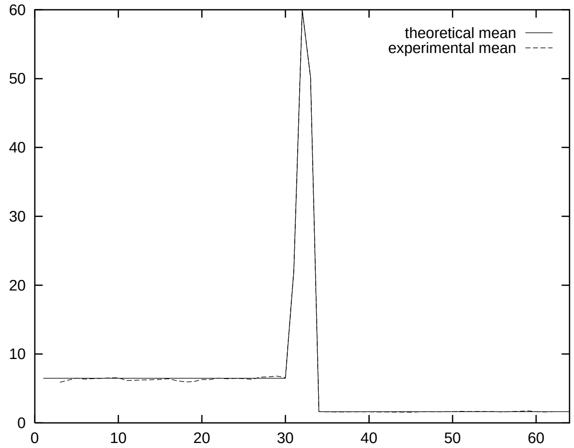

Figure 1: Experimental and theoretical mean of the response of a CQR detector of order $i = 1$ and $N = 5$ on an edge corrupted by impulsive noise.

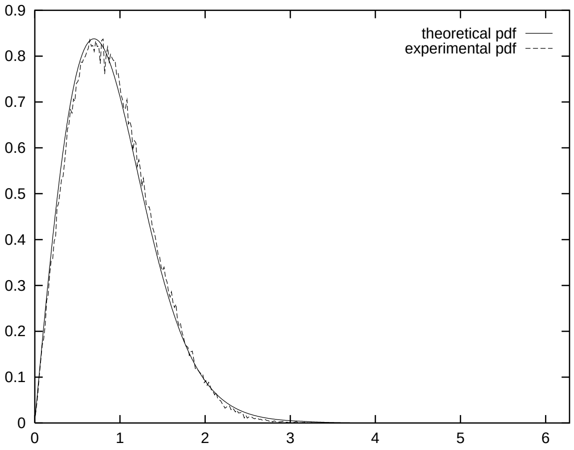

Figure 2: Experimental and theoretical output pdf for the MCQR of length $N = 3$ and order $i = 0$ applied on a constant signal of value π *rad* corrupted by $k = 4$, $\mu_0 = 0$ von Mises noise.

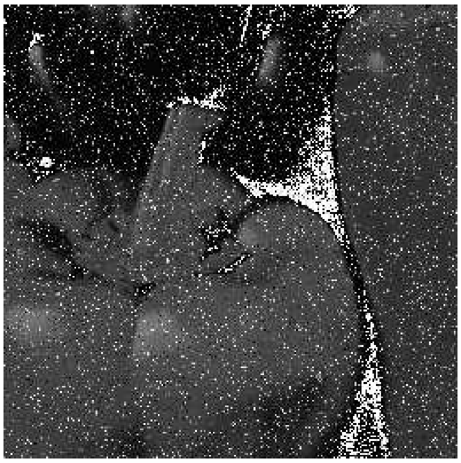

Figure 3: Hue component of test image "Pepper" corrupted by 10% impulsive noise acting on hue and having uniformly distributed impulses.

Figure 4: Output of 5×5 MCQR edge detector of order $i = 4$ applied on the noisy hue component.

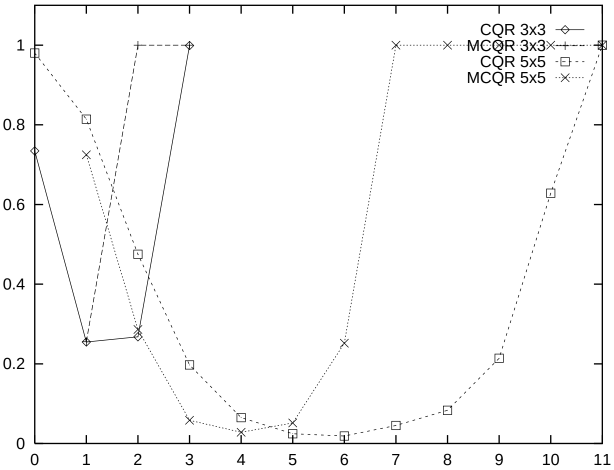

Figure 5: Figure of merit R_{min} for CQR and MCQR angular edge detectors of window size 3×3 and 5×5 applied on a noisy hue edge, for various orders i. The hue edge resulted from the HSV transformation of an RGB edge corrupted by impulsive noise having uniformly distributed value and probability of occurrence $p = 10\%$ on each RGB channel.

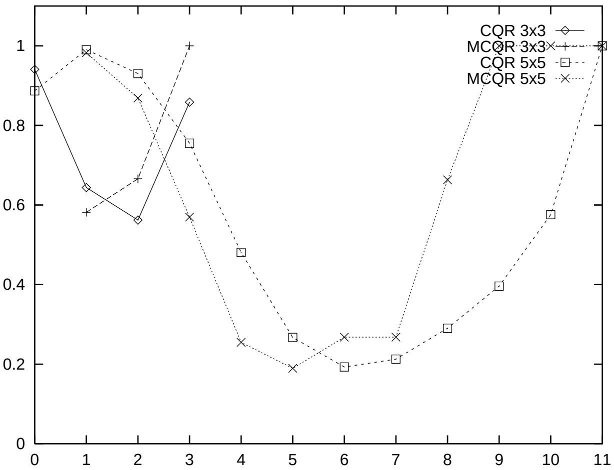

Figure 6: Figure of merit R_{min} for CQR and MCQR angular edge detectors of window size 3×3 and 5×5 applied on a noisy hue edge, for various orders i. The hue edge resulted from the HSV transformation of an RGB edge corrupted by impulsive noise having uniformly distributed value and probability of occurrence $p = 20\%$ on each RGB channel.