Morphological Signal Decomposition

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ABSTRACT — **The paper deals with the representations of signals and images by using mathematical morphology. The problem is related to multiscale analysis and is of considerable interest in computer vision, signal (image) analysis, coding, etc. A multiple structural functions decomposition algorithm based on morphological shape decomposition is proposed. The algorithm produces more compact representations than the original morphological shape decomposition technique. Experimental results are provided.**

1 Introduction

We address the problem of the representation of signals and images in terms of simple patterns characterized by their shape and size. The shape-size relation is reflected by the scale concept [1]. Since mathematical morphology is well-known for its ability to extract information on shape, it is a good candidate for such representations. Besides, scale relates naturally to the size of the structuring function. A classical morphological representation scheme is the skeleton [2]. A different representation is morphological shape decomposition [3]. It recursively decomposes binary images in components of maximal scale. Unlike in skeletonization, each component is computed from the residual image, i.e., the difference between the original image and the union of previously determined components. Morphological shape decomposition produces more compact representations than the skeleton.

This paper proposes a modified decomposition scheme based on morphological shape decomposition. The outline of the paper is as follows. The morphological signal decomposition technique is presented in section 2. The modified scheme is proposed in section 3. Experimental results are shown in section 4 and conclusions are drawn in section 5.

2 Morphological signal decomposition

A graylevel extension for morphological shape decomposition was given in [4]. We shall use the basic notations and operations for grayscale morphology as defined in [5]. In what follows we consider sampled images and signals defined over the Euclidean grid Z^n. Let $f(x)$ be a grayscale image defined on $D \subset Z^2$ and $g(x)$ be a structuring function. Without any loss of generality, we assume positively defined signals, i.e. $f(x) \geq 0$. Furthermore, we consider structuring functions satisfying $g(0) > 0$, so that erosions and openings are antiextensive transforms. Finally, the support D' of the erosion $[f \ominus g^s](x)$ is limited to those points $x \in D$ for which $[f \ominus g^s](x) \geq 0$

The main idea of morphological signal decomposition is to decompose the image into a sum of components using the tools of mathematical morphology. The components are of the form $[l_i \oplus ng](x)$ where ng is the structuring function dilated n times by itself:

$$ng(x) = [g \oplus g \oplus \ldots \oplus g](x) \tag{1}$$

The functions l_i, "loci" functions, represent the loci of the centers of the maximal structuring function in the current residual image. They are positive over their domain. The components are maximal functions in the sense that they describe the largest scaled version of the structuring function $g(x)$ that can still be fitted inside the current residue. The maximal structuring function is an invariant for the residue. The maximal function ng of a structuring function g in an image f is defined as:

$$\begin{aligned} \forall x \in D, &\quad f(x) \ominus (n+1)g(x) < 0, \\ \exists x \in D, &\quad f(x) \ominus ng(x) > 0 \end{aligned} \tag{2}$$

The decomposition is recursive. In each step, a maximal component, f_i, is computed from the current residual image and the residual image is updated for the next step. The procedure is defined by the following formulae:

$$f_i(x) = [[(f - f'_{i-1}) \ominus n_i g^s] \oplus n_i g](x) \quad (3)$$

$$f'_i(x) = \sum_{j=0}^{i} f_j(x) \quad (4)$$

$$f'_0(x) = 0 \quad (5)$$

The functions, $f'_1(x), f'_2(x), \ldots, f'_n(x), \ldots,$ are representations of $f(x)$ at increasing levels of detail. The reconstruction procedure consists of the computation of $f'_i(x)$, which means the computation of all its components $f_1(x), f_2(x), \ldots, f_n(x)$. Thus, the representation of $f(x)$ is given by the $l_i(x)$ functions together with the size of the corresponding structuring function.

The quality of the representation depends on the shape and size of the structuring function. The details that are smaller than the structuring function are lost and the reconstructed function is a smoothed version of the original. The components of the representation are simple in the sense that the opening is a simplified (filtered) version of the original. Furthermore, their supports are multivalued functions defined over a compact support. These facts lead to an increased size of the representation.

3 Modified decomposition

Our aim is to achieve an accurate representation by using simple components and to significantly decrease its overall size. Thus, we have modified the decomposition scheme described in the previous section. The modifications are related to the structuring functions and the components [6].

The original algorithm uses only one structuring function. In order to increase the convergence speed we use a library of structuring functions. Different structuring functions will better describe different parts of the image. Let $S = \{g_1, g_2, ..., g_k\}$ be a library of structuring functions. In each step, for the current residual image, the maximal functions $n_j g_i$ corresponding to each structuring function g_i is computed. Let L be a signal norm (e.g. the L_2 norm). The structuring function that minimizes the residual image in the sense of L norm is selected, let it be g_m. We have:

$$n_j g_m(x) = \{g_m(x) \in S : \ \min L(f - f_i)\} \quad (6)$$

In order to be efficient for the image description, it is necessary that the structuring functions be well adapted to the input image, i.e. parts of the image can be locally well approximated with a small number of dilated versions of the structuring functions. The development of a library of structuring functions is a heuristic procedure that requires practical experience and intuition. A large library has better chances to efficiently describe several images but the computational complexity increases prohibitively. The idea of using multiple structuring functions was also presented in [7]. We have developed a library of structuring functions inspired by the main features that exist in natural images like plateaux, peaks, slanted regions, edges. Examples of some structuring functions are shown in Fig. 1.

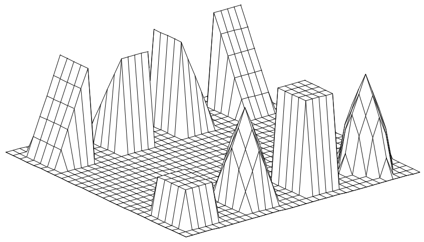

Figure 1: Structuring functions.

The components of the representations are overlapped scaled structuring functions. Their support is compact. The number of centers of each component ranges from several points to hundreds and even thousands of points. Most of these points are redundant or have a small effect in the reconstruction. It is natural to investigate the redundancy reduction of the support of the components. The l_i functions are skeletal subsets of the current residue. The concept of *minimal skeleton* was defined as the subset of the skeleton containing no redundant points, from which perfect reconstruction of the original image is possible. Several algorithms for minimal skeleton were presented (see [2], [8]). However, in our case we operate only on a single subset of the skeleton and there is no imperative requirement for an exact reconstruction. We have to mention also that no optimal algorithm is available for skeleton reduction. Under these circumstances, our approach is to select for each component only the centers of the nonoverlapping structuring function fitted beneath the current residue. We have investigated several procedures for the selection of the cen-

Figure 2: Original image.

ters: selection of the center that minimizes given criteria or norms (L_2, entropy, etc.), selection of the first center in a given order, etc. The most convenient for its simplicity is the selection of the first center, considering all the points of the support D as being ordered by an ordering relation, i.e. the lexicographic ordering. The components generated by this procedure are homothetics of the maximal structuring function. Thus, it eliminates the redundancy due to the overlap of the structuring functions but does not guarantee a minimal number of components. The method is also sensitive to the ordering principle used.

The loci are computed by successive erosions of the residual image:

$$l_i(x) = [\ldots[(f - f'_{i-1}) \ominus g^s] \ldots \ominus g^s](x) \qquad (7)$$

Since l_i is a multivalued function, it is necessary to store for each component its values and its coordinates. With a little loss in convergence speed (but with a gain in compression) we consider l_i as univalued functions with constant value 1 over the entire domain. This simplification eliminates the necessity of storing and coding the support values.

4 Experimental results

We developed a library of structuring functions for image representations. Several types of structuring functions were considered, e.g. flat squares, tilted squares and pyramids. The sizes of their supports were 3×3, 5×5, 7×7 and 9×9, and their heights ranged between 1 to 50.

Table 1: Modified decomposition performances with respect to the number of spine centers.

No. of centers	PSNR	Compression Ratio
10000	28.99	16.15
7000	28.16	21.56
6000	27.35	24.04
5000	26.89	28.05
4000	25.20	34.00
3000	24.13	42.88
2000	22.73	60.00
1000	20.07	110.03

Experimentally, we selected 32 structuring functions. The experiments were performed on real images.

The decomposition method was tested on several images. We present the results of the tests on the image "Lena" (Fig. 2) of size 512×512 at 8 bits per pixel. The representation of the image is given by the information needed to reconstruct the decomposition components, i.e. the label of the structuring function, the coordinates (the center) of the component, and the number of dilations for each component. Thus, by using the original morphological signal decomposition algorithm 132,000 coordinates of the loci functions are necessary to be stored. A square structuring function of size 2×2 and height 1 was used in the decomposition. By using our approach, only 10,000 simple components are necessary for quite similar results. Quantitatively, this means 10,000 coordinates of the loci functions plus the label and the size of the corresponding structuring function. Reconstructed images by using different number of components are presented in Fig. 3. The results are of interest for image compression applications. 27 out of 32 structuring functions have been used in the decomposition. The quality of the reconstruction was evaluated by computing the Peak-to-peak Signal to Noise ratio (PSNR). Huffman coding was further used to compute the compression ratio for each reconstructed image. Our results are summarized in Table 1.

5 Conclusions

Morphological signal decomposition is a useful representation scheme for multiscale signal and image analysis. A modified decomposition algorithm has been proposed. The components of the new representation are simple functions with discrete support. They are homothetics of the structuring functions. A library of structuring functions for image decomposition has been developed. The new method is of interest for lossy graylevel image compression, for progressive coding and transmission. The experiments show that, even at high compression ratios (between 50 and 100) the reconstructed images preserve most of the features of the original image.

The method is suitable for an efficient implementation on a dedicated morphological machine. The decomposition algorithm is easy parallelizable. The computational effort of the method is mainly in the decomposition scheme; the reconstruction procedure is fast.

Figure 3: Reconstructed images: 1,000 components (top), 2,000 components (middle), 7,000 components (bottom)

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