FAST MULTISCALE MATHEMATICAL MORPHOLOGY FOR FRONTAL FACE AUTHENTICATION

Constantine Kotropoulos * *Attila Fazekas* ** *and Ioannis Pitas* *
* Department of Informatics, Aristotle Univ. of Thessaloniki, 546 21 Thessaloniki, Greece
{costas,pitas}@zeus.csd.auth.gr
**Institute of Mathematics and Informatics, Univ. of Debrecen, H-4010 Debrecen, Hungary
fattila@math.klte.hu

ABSTRACT

Elastic graph matching based on multiscale morphological dilation-erosion was proved very efficient in terms of its receiver operating characteristic curve for frontal face authentication. In previous works, we dealt with either running (i.e., space-recursive) algorithms or scale-recursive ones. In this paper, we study the computational complexity of algorithms that combine both space and scale recursions. We derive analytically space and scale recursive algorithms for two separable structuring functions, namely the flat and the circular paraboloid. We demonstrate that for a flat structuring function such a combined scheme requires only three comparisons per pixel and scale increment.

1 INTRODUCTION

Face authentication is one of the most popular biometric verification techniques [1]. Over the last twenty years, numerous algorithms have been proposed for face recognition [2]. A powerful face recognition technique, whose origin can be traced back in the neural network community, is the dynamic link architecture (DLA) [3]. A simplified implementation of dynamic link architecture, the so-called elastic graph matching (EGM), is often preferred for locating objects in a scene with a known reference [4]. EGM algorithms that employ either multiscale dilation-erosions [5] followed by linear projections of the feature vectors at the graph nodes or morphological signal decomposition [6] were proposed in [7, 8].

In the previous works [7, 8], our prime concern was to demonstrate the authentication capabilities of the variants of EGM that were based on mathematical morphology operations in terms of their receiver operating characteristic curves (i.e., their false acceptance and false rejection rates). Although running algorithms for multiscale morphology with a flat or a circular paraboloid structuring function as well as a scale-recursive algorithm for a flat structuring function were developed and tested in [7], the computational complexity of these algorithms is still high. In this paper, we develop algorithms that exploit both recursion types in order to reduce further the number of comparisons needed for the computation of the feature vectors at the elastic graph nodes. More specifically, we derive analytically space and scale recursive algorithms for two separable structuring functions, the flat and the circular paraboloid. We demonstrate that for a flat structuring function such a combined scheme requires only three comparisons per pixel and scale increment. Such a computational complexity reduction is beneficial to low-power multiscale morphological co-processors for mobile face authentication devices [9, 10].

The outline of the paper is as follows. Section 2 describes the running and scale-recursive approaches for multiscale mathematical morphology operations. A scale-recursive algorithm for grayscale dilation with a circular paraboloid structuring function is developed for completeness. Analytical results for the proposed combined approach are presented in Section 3. A discussion on the benefits of the algorithms developed in this paper is made in Section 4. Experimental results are presented in Section 5.

2 RUNNING AND SCALE-RECURSIVE ALGORITHMS FOR MULTISCALE MATHEMATICAL MORPHOLOGY

An alternative to linear techniques used for generating an information pyramid is the scale-space morphological techniques. Multiscale dilation-erosion [5] could substitute the Gabor wavelet transform usually employed to extract local features at a given image pixel. Among the reasons that justify such a substitution is that dilations and erosions deal with the local extrema in an image. Therefore, they are well-suited for facial feature representation, because key facial features are associated to either local maxima (e.g. the nose tip) or local minima (e.g. eyebrows/eyes, nostrils, endpoints of lips etc.)

The multiscale morphological dilation-erosion is based on the two fundamental operations of grayscale morphology, namely the dilation and the erosion. Let $\mathbb{R}$ and $\mathbb{Z}$ be the set of real and integer numbers, respectively. Let us denote by boldface letters two-dimensional (2-D) pixel coordinates, i.e. $\mathbf{x} = (x_1, x_2)$. Given an image $f(\mathbf{x}) : \mathcal{D} \subseteq \mathbb{Z}^2 \to \mathbb{R}$ and a structuring function $g(\mathbf{x}) : \mathcal{G} \subseteq \mathbb{Z}^2 \to \mathbb{R}$, the dilation of the image $f(\mathbf{x})$ by $g(\mathbf{x})$ and its complementary operation, the erosion, are denoted by $(f \oplus g)(\mathbf{x})$ and $(f \ominus g)(\mathbf{x})$, respectively. Their definition can be found in [11]. If the structuring function is chosen to be scale-dependent, that is $g_\sigma(\mathbf{z}) = |\sigma| g(|\sigma|^{-1}\, \mathbf{z})$ $\forall \mathbf{z} \in \mathcal{G}$: $\|\mathbf{z}\| \leq |\sigma|$, the morphological operations become scale-dependent as well. In this paper, we deal with the flat structuring function [11]

$$g_\sigma(\mathbf{z}) = 0 \tag{1}$$

and the circular paraboloid [12]

$$g_\sigma(\mathbf{z}) = -|\sigma| \frac{\|\mathbf{z}\|^2}{\sigma^2} \tag{2}$$

where $\|\mathbf{z}\| \leq |\sigma|$.

The multiscale dilation-erosion of the image $f(\mathbf{x})$ by $g_\sigma(\mathbf{x})$ is defined as follows [5]:

$$(f \star g_\sigma)(\mathbf{x}) = \begin{cases} (f \oplus g_\sigma)(\mathbf{x}) & \text{if } \sigma > 0 \\ f(\mathbf{x}) & \text{if } \sigma = 0 \\ (f \ominus g_{|\sigma|})(\mathbf{x}) & \text{if } \sigma < 0. \end{cases} \tag{3}$$

The outputs of multiscale dilation-erosion for several scales $\sigma = -\sigma_m, \ldots, \sigma_m$ form a local feature vector. A common choice for frontal face authentication experiments is $\sigma_m = 9$ [7].

It has been found that the choice of the structuring function affects the authentication capability of the proposed technique to a margin of $\pm 0.5\%$ with respect to the equal error rate, but it does affect the time required to compute the dilation and erosion [7]. The 2-D flat structuring function and the circular paraboloid one are separable, because they can be decomposed in terms of the one-dimensional (1-D) structuring function $g_\sigma^{(1)}(z_i)$, $i = 1, 2$, i.e.,

$$g_\sigma(z_1, z_2) = g_\sigma^{(1)}(z_1) + g_\sigma^{(1)}(z_2). \tag{4}$$

Our interest will be confined to separable structuring functions. Two classes of algorithms are considered, namely the running (i.e., space-recursive) algorithms and the scale-recursive ones. Throughout the paper, we study the computation of grayscale dilation. However, the results can easily be extended to the computation of grayscale erosion.

2.1 Running algorithms

For a flat structuring function, dilations can be efficiently computed by applying running max calculations (e.g., the MAXLINE algorithm [13]) in which the computation of $(f \oplus g_\sigma)(x_1, x_2)$ exploits the previous outcome $(f \oplus g_\sigma)(x_1 - 1, x_2)$. Other running max algorithms are described in [14, 15]. Similar running min calculations can be exploited in the efficient computation of erosions.

Let us consider next a 2-D circular paraboloid structuring function. It can easily be seen that [12]

$$(f \oplus g_\sigma)(x_1, x_2) = \max_{z_1 \in \mathcal{G}_\sigma^{(1)}} \Big(\gamma(x_1 - z_1, x_2) + g_\sigma^{(1)}(z_1) \Big) \tag{5}$$

$$\gamma(x_1, x_2) = \max_{z_2 \in \mathcal{G}_\sigma^{(1)}} \Big(f(x_1, x_2 - z_2) + g_\sigma^{(1)}(z_2) \Big) \tag{6}$$

where $\mathcal{G}_\sigma^{(i)}$, $i = 1, 2$ are the projections of $\mathcal{G}$ on the two axes. Following similar lines as in [12], let us suppose that the maximum occurs in (6) for $z_2 = \xi$. Then

$$\gamma(x_1, x_2{+}1) = \max_{z_2 = -\sigma}^{\xi+1} \Big(f(x_1, x_2 + 1 - z_2) + g_\sigma^{(1)}(z_2) \Big). \tag{7}$$

Eq. 7 is the basis of a running separable implementation of grayscale dilation with a 2-D circular paraboloid structuring function.

2.2 Scale-recursive algorithms

For a flat structuring function, scale-recursive max computations are based on the observation that

$$\begin{aligned} (f \oplus g_{\sigma+1})(x_1, x_2) = \max\Big\{ & (f \oplus g_\sigma)(x_1, x_2), \\ & \max_{(z_1, z_2) \in \Delta G(\sigma+1)} \{ f(x_1 + z_1, x_2 + z_2) \}, \\ & \max\{ f(x_1 \pm (\sigma+1), x_2 \pm (\sigma+1)) \} \Big\} \end{aligned} \tag{8}$$

where the set $\Delta G(\sigma + 1) = \{(z_1, z_2) \in \mathcal{Z}^2 : (z_1^2 + z_2^2) > \sigma^2,\ (z_1^2 + z_2^2) \leq (\sigma+1)^2,\ |z_1| \leq \sigma,\ |z_2| \leq \sigma\}$ possesses a symmetry and can easily be evaluated prior to the computation of dilations.

Next, we derive analytically a scale-recursive algorithm for the grayscale dilation when a circular paraboloid structuring function (2) is employed. For the 1-D paraboloid, we have

$$g_{\sigma+1}(z) = \frac{\sigma}{\sigma+1} g_\sigma(z), \quad |z| \leq \sigma \tag{9}$$

$$g_{\sigma+1}(z) \geq g_\sigma(z), \quad |z| \leq \sigma. \tag{10}$$

Let us explicitly indicate the scale parameter that corresponds to the computation of $\gamma(x_1, x_2)$ defined by (6) with the addition of a subscript. It can easily be shown that a recursive computation of $\gamma_{\sigma+1}(x_1, x_2)$ is given by

$$\gamma_{\sigma+1}(x_1,x_2) = \max\Big(\max_{|z_2|\le\sigma} \{ f(x_1, x_2 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2)\}, \max_{\sigma<|z_2|\le(\sigma+1)} \{ f(x_1, x_2 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2)\} \Big). \quad (11)$$

Let us suppose that the maximum in $\gamma_\sigma(x_1, x_2)$ occurs for $z_2 = \xi \in [-\sigma, \sigma]$. That is,

$$f(x_1, x_2-\xi) + g_\sigma(\xi) \ge f(x_1, x_2 - z_2) + g_\sigma(z_2), \quad |z_2| \le \sigma. \quad (12)$$

It can be proven that

$$\max_{|z_2|\le|\xi|} \{ f(x_1, x_2 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2)\} = \gamma_\sigma(x_1,x_2) - \frac{1}{\sigma+1} g_\sigma(\xi). \quad (13)$$

Accordingly,

$$\gamma_{\sigma+1}(x_1,x_2) = \max\Big([\gamma_\sigma(x_1,x_2) - \frac{1}{\sigma+1} g_\sigma(\xi)], \max_{\substack{z_2\in[-\sigma-1,\,-|\xi|) \\ \cup(|\xi|,\,\sigma+1]}} \{ f(x_1, x_2 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2)\} \Big). \quad (14)$$

3 COMBINED APPROACH FOR SEPARABLE STRUCTURING FUNCTIONS

In this section, we derive analytically combined space and scale recursive algorithms for grayscale dilation. When a flat structuring function is used, then the max computations can be rearranged as follows:

$$(f \oplus g_{\sigma+1})(x_1, x_2) = \max_{\substack{|z_1|\le\sigma+1 \\ |z_2|\le\sigma+1}} \Big\{ f(x_1 - z_1, x_2 - z_2) \Big\}$$

$$= \max_{|z_1|\le\sigma+1} \Big\{ \max_{|z_2|\le\sigma} \{ f(x_1 - z_1, x_2 - 1 - z_2)\}, \max_{|z_2|\le\sigma} \{ f(x_1 - z_1, x_2 + 1 - z_2)\} \Big\}$$

$$= \max\Big\{ (f\oplus g_\sigma)(x_1 - 1, x_2 - 1), (f\oplus g_\sigma)(x_1 - 1, x_2 + 1), (f\oplus g_\sigma)(x_1 + 1, x_2 - 1), (f\oplus g_\sigma)(x_1 + 1, x_2 + 1) \Big\}. \quad (15)$$

Subsequently, we develop a space and scale recursive algorithm for a grayscale dilation, when a circular paraboloid structuring function is employed. Eq. (11) can be rewritten as

$$\gamma_{\sigma+1}(x_1,x_2) = \max\Big(\max_{|z_2|\le\sigma} \{ f(x_1, x_2 + 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 - 1)\}, \max_{|z_2|\le\sigma} \{ f(x_1, x_2 - 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 + 1)\} \Big). \quad (16)$$

It can be shown that

$$\max_{|z_2|\le\sigma} \{ f(x_1, x_2 + 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 - 1)\} = \max_{|z_2|\le\sigma} \{ f(x_1, x_2 + 1 - z_2) + g_\sigma(z_2) - \frac{1}{\sigma+1} \cdot [g_\sigma(z_2) + 1 - 2z_2]\}. \quad (17)$$

Let us suppose that the maximum in $\gamma_\sigma(x_1, x_2+1)$ occurs for $z_2 = \xi_1 \in [-\sigma, \sigma]$. That is, for $|z_2| \le \sigma$, the following inequality holds:

$$f(x_1, x_2+1-\xi_1) + g_\sigma(\xi_1) \ge f(x_1, x_2+1-z_2) + g_\sigma(z_2). \quad (18)$$

Using (18), the maximum in (17) can be computed by

$$\max_{|z_2|\le\sigma} \{ f(x_1, x_2 + 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 - 1)\} = \max\Big(\gamma_\sigma(x_1, x_2+1) - \frac{1}{\sigma+1}[g_\sigma(\xi_1) + 1 - 2\xi_1], \max_{z_2=\xi_1+1}^{\sigma} \{ f(x_1, x_2 + 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 - 1)\} \Big). \quad (19)$$

Similarly, we obtain

$$\max_{|z_2|\le\sigma} \{ f(x_1, x_2 - 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 + 1)\} = \max_{|z_2|\le\sigma} \{ f(x_1, x_2 - 1 - z_2) + g_\sigma(z_2) - \frac{1}{\sigma+1} \cdot [g_\sigma(z_2) + 1 + 2z_2]\}. \quad (20)$$

Let us suppose that the maximum in $\gamma_\sigma(x_1, x_2-1)$ occurs for $z_2 = \xi_2 \in [-\sigma, \sigma]$. That is, for $|z_2| \le \sigma$, we have

$$f(x_1, x_2-1-\xi_2) + g_\sigma(\xi) \ge f(x_1, x_2-1-z_2) + g_\sigma(z_2). \quad (21)$$

Then, (20) can be computed recursively using

$$\max_{|z_2|\le\sigma} \{ f(x_1, x_2 - 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 + 1)\} = \max\Big(\gamma_\sigma(x_1, x_2-1) - \frac{1}{\sigma+1}[g_\sigma(\xi_2) + 1 + 2\xi_2], \max_{z_2=-\sigma}^{\xi_2-1} \{ f(x_1, x_2 - 1 - z_2) + \frac{\sigma}{\sigma+1} g_\sigma(z_2 + 1)\} \Big). \quad (22)$$

The combination of (16), (19), and (22) constitutes the heart of a space and scale-recursive algorithm for the computation of $\gamma(x_1, x_2)$.

4 DISCUSSION

As a basis of our discussion, we consider the fact that the maximum of n numbers requires at most $2\log_2 n$ comparisons [13]. For all algorithms an upper bound on the number of comparisons can be estimated and this bound is used as figure of merit. For the flat structuring function, the discussion refers to the computation of the grayscale dilation, whereas for the circular paraboloid the discussion refers to the computation of $\gamma(x_1, x_2)$. In this case, the grayscale dilation is obtained by combining (5) and (6).

For a flat structuring function and for sufficiently large σ, the MAXLINE algorithm requires *on average* 3 comparisons to compute $(f \oplus g_\sigma)(x_1, x_2)$, if the probability density function of the gray level values is uniform [13]. A straightforward computation of the gray scale dilation would require $4\log_2(2\sigma+1)$.

The straightforward computation of $\gamma(x_1, x-2)$ requires $2\log_2(2\sigma+3)$ comparisons. The number of comparisons needed to compute (7) is roughly $2\log_2(\xi+\sigma+2)$ which is less than that required for the straightforward computation of $\gamma(x_1, x_2+1)$, if $\xi < \sigma - 1$.

A rough estimate of the number of comparisons needed to compute $(f \oplus g_{\sigma+1})(x_1, x_2)$ in (8) is $2\log_2(8\sigma+9)$, which is always less than that required for the straightforward computation, i.e., $4\log_2(2\sigma+3)$.

Eq. (14) requires $2\log_2(2\sigma - 2\xi + 5)$ comparisons. The number of comparisons in a scale-recursive computation is less than that required for the computation of $\gamma_\sigma(x_1, x_2+1)$ by a running algorithm, if $|\xi| > \lceil \frac{\sigma}{3} + 1 \rceil$. Moreover, the number of comparisons in (14) is less than the number of $2\log_2(2\sigma+3)$ comparisons required by the straightforward computation, if $|\xi| > 1$.

For a flat structuring function, (15) demonstrates that only 3 comparisons are needed per scale increment, when both space and scale recursions are exploited. There is a clear reduction in computations with respect to scale recursive algorithms. Moreover, the number of comparisons is now *exact* and not an average one.

From the inspection of the recursions (19) and (22) becomes evident that the space and scale recursive computation of $\gamma_{\sigma+1}(x_1, x_2)$ requires $2\log(2\sigma - \xi_1 + \xi_2 + 2)$ comparisons. This number is less than the number of $2\log_2(2\sigma+3)$ comparisons required by the straightforward computation, if $\xi_1 - \xi_2 > 1$.

5 EXPERIMENTAL RESULTS

We implemented all the proposed algorithms for grayscale dilation and erosion, when a flat structuring function is being used. We are interested in comparing the time needed for the computation of the multiscale dilation-erosion for $\sigma = -9, \ldots, 9$ that is used in the frontal face verification algorithm described in [7], when these algorithms are being exploited. Implementations of grayscale dilation and erosion using the scaled hemisphere and the circular paraboloid (either separable or running) were included in our study as well. Figure 1 depicts the time needed to compute the multiscale dilation-erosion for typical facial images of dimensions 286×350 pixels from the M2VTS database [16]. A set of 37, one per person, frontal faces was used. The computations were performed on a SUN Ultra10 workstation with UltraSPARC-IIi processor at 300 MHz and 256 MB RAM. The average computation time and the standard deviation of the computation time over the set of the 37 frontal facial images for each implementation are summarized in Table 1. As a result, when

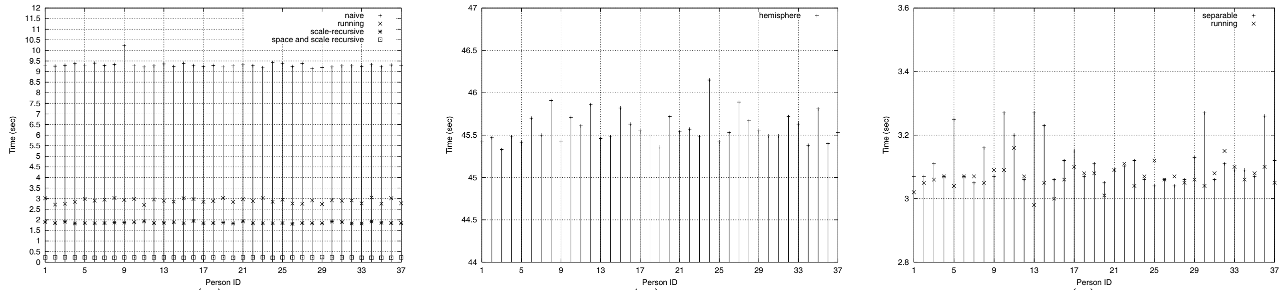
Table 1: Average and standard deviation of the computation time (in sec) for each implementation of multiscale dilation-erosion.

Structuring function	Algorithm	Average Time	Standard deviation
Flat	Naive	9.311	0.168
	Running	2.898	0.101
	Scale-recursive	1.864	0.035
	Space & scale recursive	0.219	0.007
Scaled-hemisphere	Straightforward	45.584	0.181
Circular paraboloid	Separable	3.116	0.073
	Running	3.068	0.035

both space and scale recursions are applied to multiscale dilation-erosion, the average time for a single frontal face verification drops from 5.218 sec, measured with scale recursive mathematical morphology operations, to 3.565 sec. The latter average time refers to the graph matching algorithm.

Acknowledgement

This work has been supported by the European Union Research Training Network "Multimodal Human-Computer Interaction" (HPRN-CT-2000-00111).



(a) (b) (c)

Figure 1: Computation time for multiscale dilation-erosion for 37, one per person, frontal facial images from the M2VTS database for (a) flat, (b) scaled hemisphere, and (c) circular paraboloid structuring functions.

References

[1] J.-L. Dugelay, J.-C. Junqua, C. Kotropoulos, R. Kuhn, F. Perronnin, and I. Pitas, "Recent advances in biometric person authentication," in *Proc. 2002 IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, vol. IV, pp. 4060-4063, Florida, 2002.

[2] R. Chellappa, C. L. Wilson, and S. Sirohey, "Human and machine recognition of faces: A survey," *Proceedings of the IEEE*, vol. 83, no. 5, pp. 705-740, May 1995.

[3] M. Lades, J. C. Vorbrüggen, J. Buhmann, J. Lange, C. v.d. Malsburg, R. P. Würtz, and W. Konen, "Distortion invariant object recognition in the Dynamic Link Architecture," *IEEE Trans. on Computers*, vol. 42, no. 3, pp. 300-311, March 1993.

[4] J. Zhang, Y. Yan, and M. Lades, "Face recognition: Eigenface, elastic matching and neural nets," *Proceedings of the IEEE*, vol. 85, no. 9, pp. 1423-1435, September 1997.

[5] P. T. Jackway and M. Deriche, "Scale-space properties of the multiscale morphological dilation-erosion," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 18, no. 1, pp. 38–51, January 1996.

[6] I. Pitas and A. N. Venetsanopoulos, "Morphological shape decomposition," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 12, no. 1, pp. 38-45, January 1990.

[7] C. Kotropoulos, A. Tefas, and I. Pitas, "Frontal face authentication using discriminating grids with morphological feature vector," *IEEE Trans. Multimedia*, vol. 2, no. 1, pp. 14-26, March 2000.

[8] A. Tefas, C. Kotropoulos, and I. Pitas, "Face verification using elastic graph matching based on morphological signal decomposition," *Signal Processing*, vol. 8, no. 6, pp. 833-851, June 2002.

[9] P. Stadelmann, J.-L. Nagel, M. Ansorge, and F. Pellandini, "A multiscale morphological coprocessor for low-power face authentication," in *Proc. XI European Signal Processing Conf.*, vol. III, pp. 581-584, Toulouse, France.

[10] J.-L. Nagel, P. Stadelmann, M. Ansorge, and F. Pellandini, "A low-power VLSI architecture for face verification using elastic graph matching," in *Proc. XI European Signal Processing Conf.*, vol. III, pp. 577-580, Toulouse, France.

[11] I. Pitas and A. N. Venetsanopoulos, *Nonlinear Digital Filters: Principles and Applications.* Norwell, MA: Kluwer, 1990.

[12] P. T. Jackway, "Morphological scale-space with application to three-dimensional object recognition," Ph.D. Dissertation, Queensland University of Technology, Australia, 1994.

[13] I. Pitas, "Fast algorithms for running ordering and max/min calculation," *IEEE Trans. on Circuits and Systems*, vol. CAS-36, no. 6, pp. 795-804, June 1989.

[14] S. Douglas, "Running max/min calculation using a pruned ordered list," *IEEE Trans. on Signal Processing*, vol. 44, no. 11, pp. 2872-2877, November 1996.

[15] D. Z. Gevorkian, J. T. Astola, and S. M. Atourian, "Improving Gil-Werman algorithm for running min and max filters," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 19, no. 5, pp. 526-529, May 1997.

[16] S. Pigeon and L. Vandendorpe, "The M2VTS multimodal face database," in *Lecture Notes in Computer Science: Audio- and Video- based Biometric Person Authentication* (J. Bigün, G. Chollet and G. Borgefors, Eds.), vol. 1206, pp. 403-409, 1997.