Nonlinear filtering of speckle noise in ultrasonic images

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1 Introduction

Speckle noise is a special kind of noise encountered in images formed by laser beams, in radar images as well as in envelope-detected ultrasound (US) B-mode images. It is an interference effect caused by the scattering of the US beam from microscopic tissue inhomogeneities. It has been found that the contrast/detail results for the envelope detection in diagnostic US are almost identical with the results for square law detection with the latter serving as upper limit for performance in lesion detection [6]. The detection of focal lesions from the point of view of communication systems has been considered in [7]. The suppression of speckle by an adaptive weighted median filter has been proposed in [3].

The main contribution of this paper is the design of optimal nonlinear filters for speckle removal in US B-mode images and the derivation of their properties. Speckle is modeled as multiplicative noise. At a first approach the signal is assumed to be constant and the noise term to be Rayleigh random variable having unity expected value. The detection of the constant signal is expressed as a binary hypothesis-testing problem. The receiver operating characteristics for the optimal decision rule are derived by evaluating theoretically the probability of false alarm and the probability of detection. The problem of estimating the constant signal is also considered. It is proven that the maximum likelihood (ML) estimator of the signal is the L_2 mean filter multiplied by a constant scaling factor. The expected value and the variance of this estimator have been evaluated. The mean square error (MSE) in estimating the constant signal by using the ML-estimator has also been calculated. The use of an L-estimator of the constant signal is also proposed. L-estimators are defined as linear combinations of the order statistics, i.e., the observations arranged in ascending order of their magnitude inside the filter window [1]. The L-estimator which minimizes the mean square error between the L-estimator output and the signal is designed. At a second approach, the signal is assumed to be random variable. The structure of the optimal decision rule is again derived. The maximum a posteriori probability (MAP) estimator of the intensity (i.e., squared) signal has also been found.

2 Detection of a constant signal from speckle

Let z be the envelope-detected observed signal, m be the signal and n be a noise term statistically independent of m. It is assumed that the signal m is related to the observation z by:

$$z = mn \tag{1}$$

The probability density function (pdf) of the observed random variable (r.v.) z is considered to be Rayleigh [4]:

$$f_z(z) = \frac{z}{\sigma^2}\exp[-\frac{z^2}{2\sigma^2}], \quad z > 0 \tag{2}$$

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It can be easily proven that if the signal m is constant and equals $\sigma\sqrt{\pi/2}$ and the noise term n is Rayleigh r.v. having unity expected value, then the pdf of the r.v. z is given by (2). In the following, the model (1) will be used.

Let us assume that we have a set of N observations $z_1, z_2, \ldots, z_N$ denoted by a vector $\mathbf{z} = (z_1, z_2, \ldots, z_N)^t$ in the observation space $\mathcal{R}^N$. Let $\mathbf{n} = (n_1, n_2, \ldots, n_N)^t$ be a vector of N independent identically distributed Rayleigh noise random variables. Let us assume the following two hypotheses:

$$H_k : \mathbf{z} = m_k \mathbf{n} \qquad k = 0, 1 \tag{3}$$

created by the probabilistic transition mechanisms:

$$f_{z_i|H_k}(Z_i|H_k) = \frac{Z_i}{\sigma_k^2}\exp[-\frac{Z_i^2}{2\sigma_k^2}] \quad Z_i > 0, \ \ i = 1, \ldots, N \ \ k = 0, 1 \tag{4}$$

The Bayes criterion [8] leads to the likelihood ratio test (LRT) :

$$\Lambda(\mathrm{Z}) = \frac{f_{\mathbf{z}|H_1}(\mathbf{Z}|H_1)}{f_{\mathbf{z}|H_0}(\mathbf{Z}|H_0)} \overset{H_1}{>} \theta \tag{5}$$

By substituting (4) to (5) the following optimal decision rule results:

$$\sum_{i=1}^{N} Z_i^2 \overset{H_1}{>} \frac{2\sigma_0^2\sigma_1^2}{\sigma_1^2 - \sigma_0^2}(\ln\theta - 2Nln\frac{\sigma_0}{\sigma_1}) = \gamma \ \text{ for } \sigma_1^2 > \sigma_0^2 \tag{6}$$

$$\sum_{i=1}^{N} Z_i^2 \overset{H_1}{<} \frac{2\sigma_0^2\sigma_1^2}{\sigma_0^2 - \sigma_1^2}(2Nln\frac{\sigma_0}{\sigma_1} - \ln\theta) = \gamma' \ \text{ for } \sigma_1^2 < \sigma_0^2 \tag{7}$$

where θ, γ and γ' are thresholds.

Let R_0 be the decision region under the hypothesis H_0 and R_1 the corresponding decision region under the alternative hypothesis. The probability of *false alarm* and the probability of *detection* for the decision rule (6) are given by:

$$P_F = \int_{R_1} f_{\mathbf{z}|H_0}(\mathbf{Z}|H_0)d\mathbf{Z} = Pr[\sum_{i=1}^{N} Z_i^2 \geq \gamma|H_0] \tag{8}$$

$$P_D = \int_{R_1} f_{\mathbf{z}|H_1}(\mathbf{Z}|H_1)d\mathbf{Z} = Pr[\sum_{i=1}^{N} Z_i^2 \geq \gamma|H_1] \tag{9}$$

The plot of P_D versus P_F for various γ as varying parameter is defined as the receiver operating characteristic. The threshold γ is expressed as follows:

$$\gamma = \frac{2d^2\sigma_0^2}{d^2 - 1}(\ln\theta + 2N\ln d) \tag{10}$$

where $d = \sigma_1/\sigma_0$. It can be proven [2] that the probability of false alarm is given by:

$$P_F = \int_{\frac{\gamma}{2\sigma_0^2}}^{\infty} \frac{\xi^{N-1}}{(N-1)!}\exp(-\xi)d\xi = 1 - \mathcal{I}_\Gamma(\frac{\gamma}{2\sigma_0^2\sqrt{N}}, N-1), \ \ d \geq 1 \tag{11}$$

where $\mathcal{I}_\Gamma(u, M)$ is the incomplete Gamma function:

$$\mathcal{I}_\Gamma(u, M) \triangleq \int_0^{u\sqrt{M+1}} \frac{x^M}{M!}\exp(-x)dx \tag{12}$$

The probability P_D is evaluated as follows:

$$P_D = 1 - \mathcal{I}_\Gamma(\frac{\gamma}{2d^2\sigma_0^2\sqrt{N}}, N-1), \ \ d \geq 1 \tag{13}$$

From (11,13) and (10) , it can be seen that the probabilities of false alarm and detection are independent of σ_0. A similar analysis holds for (7). The receiver operating characteristic becomes superior as d increases (decreases) when $d > 1$ (when $d < 1$) or with the increase of the length N when d is kept constant.

3 Estimation of a constant signal from speckle

In this section we estimate the parameter m in (1) if n is multiplicative noise independent of m which is distributed as follows:

$$f_n(\mathcal{N}) = \frac{\pi\mathcal{N}}{2}\exp[-\frac{\pi\mathcal{N}^2}{4}] \quad \mathcal{N} > 0 \tag{14}$$

Let us suppose that we have a set of N observations. Then:

$$f_{\mathbf{z}|m}(\mathbf{Z}|M) = \frac{\pi^N}{2^N M^{2N}}\prod_{i=1}^{N} Z_i \exp[-\frac{\pi Z_i^2}{4M^2}] \tag{15}$$

The ML-estimate of M maximizes the log-likelihood function $\ln f_{z|m}(\mathbf{Z}|M)$. Therefore:

$$\frac{\partial}{\partial M}\ln f_{\mathbf{z}|m}(\mathbf{Z}|M)|_{M=\hat{m}_{ML}(\mathbf{Z})} = 0 \tag{16}$$

or equivalently:

$$\hat{m}_{ML} = \frac{\sqrt{\pi}}{2}\sqrt{\frac{1}{N}\sum_{i=1}^{N} Z_i^2} \tag{17}$$

Thus, it has been proven that the ML-estimator of the constant signal is the L_2 mean scaled by the factor $\frac{\sqrt{\pi}}{2}$.

Let $\pi(N)$ be the following polynomial of N:

$$\pi(N) \triangleq \frac{\Gamma[N+\frac{1}{2}]}{\sqrt{N}(N-1)!} \tag{18}$$

then, the expected value of the ML-estimator, its variance and the mean square estimation error are given by [2]:

$$\mathrm{E}[\hat{m}] = \pi(N)M \qquad \mathrm{var}[\hat{m}] = (1-\pi^2(N))M^2 \qquad \mathrm{E}[(\hat{m}-M)^2] = 2(1-\pi(N))M^2 \tag{19}$$

Another class of estimators found extensive applications in digital signal and image processing are the L-estimators which are based on the order statistics. The output of the L-estimator of length N is given by:

$$y(k) = \mathbf{a}^t\mathbf{z}_r(k) \tag{20}$$

where $\mathbf{a} = (a_1, \ldots, a_N)^t$ is the L-estimator coefficient vector and $\mathbf{z}_r(k) = (z_{(1)}^k, z_{(2)}^k, \ldots, z_{(N)}^k)^t$ is the vector of the observations arranged in ascending order of magnitude (i.e., order statistics). We shall design the L-estimator which minimizes the mean square error (MSE) $\mathrm{E}[(y(k) - m)^2]$ under the constraint of unbiased estimation for the model (1). The unbiasedness condition implies that the L-estimator output will converge to the estimated constant signal in an ensemble-average sense and results in the following equation in vector notation:

$$\mathbf{a}^t\overline{\mu} = 1 \tag{21}$$

where $\overline{\mu} = (\mathrm{E}[n_{(1)}], \mathrm{E}[n_{(2)}], \ldots, \mathrm{E}[n_{(N)}])^t$ is the vector of the expected values of the order statistics. The superscript k is dropped out due to stationarity. Let $\mathbf{n}_r = (n_{(1)}, \ldots, n_{(N)})^t$

be the vector of the ordered noise samples and $\mathbf{R} = \mathbf{E}[\mathbf{n}_r \mathbf{n}_r^t]$ be the correlation matrix of the ordered noise samples. The MSE is written as follows:

$$MSE = m^2(\mathbf{a}^t\mathbf{R}\mathbf{a} - 1) \tag{22}$$

The L-estimator coefficient vector $\mathbf{a}$ which minimizes (22) under (21) is given by [1]:

$$\mathbf{a} = \frac{\mathbf{R}^{-1}\overline{\mu}}{\overline{\mu}^t\mathbf{R}^{-1}\overline{\mu}} \tag{23}$$

In order to calculate a we need matrix $\mathbf{R}$ whose elements are moments of the order statistics for the Rayleigh distribution with parameter σ. In our case the parameter σ equals $\sqrt{2/\pi}$, as can be seen from (14). The evaluation of the elements of the correlation matrix $\mathbf{R}$ and the vector $\overline{\mu}$ in a computationally efficient manner is treated in [2]. It has been found that the higher order statistics are weighted by larger coefficients for various L-estimator lengths and that there exists an almost linear increase in magnitude of the L-estimator coefficients $\mathbf{a}$ with the order number i.

4 Generalization to a random lesion signal

In most practical cases it is unrealistic to consider a constant signal hypothesis. Without any loss of generality the following binary hypothesis problem will be assumed:

$$\begin{aligned} H_1 : z &= mn \\ H_0 : z &= n \end{aligned} \tag{24}$$

where m, n are random variables. Our aim is to perform detection and estimation based on this model. Since m is a r.v., the conditional density of the observations assuming H_1 is given by:

$$f_{z|H_1} = \int_{\chi_m} f_{z|m,H_1}(Z|M,H_1) \;\; f_{m|H_1}(M|H_1)dM \tag{25}$$

where χ_m is the domain of the r.v. m. If n is a Rayleigh r.v. distributed as (14) then the conditional density of the observations under the hypothesis H_1 and the condition that m is known is given by:

$$f_{z|m,H_1}(Z|M,H_1) = \frac{1}{M}f_n(\frac{Z}{M}) = \frac{\pi Z}{2M^2}\exp(-\frac{\pi Z^2}{4M^2}) \quad M > 0 \tag{26}$$

The conditional density of m assuming H_1 must be chosen in such a way that it represents a realistic model and it is mathematically tractable. A Maxwell density with parameter Λ fullfills both requirements. Thus:

$$f_{m|H_1}(M|H_1) = \frac{4\Lambda^{3/2}}{\sqrt{\pi}} M^2 \exp(-\Lambda M^2) \tag{27}$$

By substituting (27) in (25) we obtain:

$$f_{z|H_1}(Z|H_1) = \pi\Lambda Z \exp(-Z\sqrt{\Lambda\pi}) \tag{28}$$

It can be seen that the resulted density is a Gamma density. Such a result is very reasonable, because it is known that speckle can be modeled by a Gamma density function [5, pp. 226]. Based on N observations the log-likelihood test leads to:

$$\sum_{i=1}^{N} Z_i^2 - 4\sqrt{\frac{\Lambda}{\pi}}\sum_{i=1}^{N} Z_i \overset{H_1}{\gtrless} \frac{4}{\pi}(\theta - N\ln 2\Lambda) = \gamma'' \quad (29)$$

The problem of the estimation of the signal m will be treated next. The (MAP) estimate of the signal is defined by:

$$\frac{\partial}{\partial M} f_{m|\mathbf{Z}}(M|\mathbf{Z})|_{M=\hat{m}_{MAP}(\mathbf{Z})} = 0 \quad (30)$$

By applying the Bayes theorem we obtain:

$$(\frac{\pi}{4}\sum_{i=1}^{N} Z_i^2)\frac{1}{M^4} - (N-1)\frac{1}{M^2} - \Lambda = 0 \quad (31)$$

Therefore the MAP estimate of the squared signal $s = m^2$ is given by:

$$\hat{s}_{MAP}(\mathbf{Z}) = \frac{\frac{\pi}{2}\sum_{i=1}^{N} Z_i^2}{(N-1) + \sqrt{(N-1)^2 + \pi\Lambda\sum_{i=1}^{N} Z_i^2}} \quad (32)$$

It can be seen that for $\Lambda = 0$ the MAP estimate of m reduces to the form of the ML-estimate of the constant signal.

5 Experimental results

In US community, simulated US B-mode images are used in order to evaluate the performance of various filters in speckle suppression and to select parameters (such as filter length and thresholds involved) in the image processing task. A simulation of an homogeneous piece of tissue (4×4 cm) with a circular lesion in the middle with diameter of 2cm has been used. The lesion differs from the background in reflection strength (+3dB). Background and lesion have an equal number density of scatterers ($5000/cm^3$). We have examined the success of the following strategies in lesion detectability: (1) thresholding the original image without any processing (2) filtering the original image by the 9×9 arithmetic mean filter and thresholding the filtered image (3) filtering the original image by the 9×9 ML-estimator of the constant signal and thresholding the filtered image (4) filtering the original image by the 9×9 L-estimator and thresholding the filtered image. We have compared the performance of the above-described strategies using as figures of merit the area under the ROC in each case and the probability of detection P_D for a threshold chosen so that the probability of false alarm P_F to be $\simeq 10\%$. Some experimental evaluations of

Table 1: FIGURES OF MERIT FOR LESION DETECTION ON SIMULATED US B-MODE IMAGE

Method	Area under ROC	$P_F(\%)$	$P_D(\%)$	Threshold
image thresholding	0.634570	10.8031	26.4860	25
ar. mean 9×9	0.738165	11.5512	37.8067	20
ML-estimator 9×9	0.743776	12.2488	40.8185	19
L-estimator 9×9	0.745246	11.5047	39.5872	19

these figures of merit are shown in Table 1. It can be seen that the proposed nonlinear filters are relatively better than the arithmetic mean with respect to the area under the ROC and the probability of detection for the same probability of false alarm.

Acknowledgements
This work has been supported by a grant from the National Scholarship Foundation of Greece and the Bodosaki Foundation.

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