Segmentation-based L-filtering of speckle noise in ultrasonic images

Eleftherios Kofidis Sergios Theodoridis

University of Patras, Department of Computer Engineering and Informatics, Patras 265 00, Greece

Constantine Kotropoulos Ioannis Pitas

University of Thessaloniki, Department of Electrical Engineering, Thessaloniki 540 06, Greece

ABSTRACT

In this paper, we introduce segmentation-based L-filters, that is, filtering processes combining segmentation and (nonadaptive) optimum L-filtering, and we use them for the suppression of speckle noise in ultrasonic (US) images. With the aid of a suitable modification of the Learning Vector Quantizer (LVQ) self-organizing neural network, the image is segmented in regions of approximately homogeneous first-order statistics. For each such region a minimum mean-squared error (MMSE) L-filter is designed on the basis of a multiplicative noise model by using the histogram of grey values as an estimate of the parent distribution of the noisy observations and a suitable estimate of the original signal in the corresponding region. Thus, we obtain a bank of L-filters that are corresponding to and are operating on different image regions. Simulation results on a simulated US B-mode image of a tissue mimicking phantom are presented which verify the superiority of the proposed method as compared to a number of conventional filtering strategies in terms of a suitably defined signal-to-noise ratio (SNR) measure and detection theoretic performance measures.

1. INTRODUCTION

Ultrasonic imaging has become an important modality in the field of medical imaging systems[1] mainly because of the nonionizing nature of the ultrasonic radiation which minimizes the risk to both patient and examiner and the unique properties of acoustical imagery which render it complementary to other diagnostic tools. However, ultrasound (US) images suffer from a special kind of noise, called "speckle", which limits the resolvability of fine details in these images and thus degrades their diagnostic value. Speckle noise is present in all kinds of coherent imagery (e.g., laser and synthetic aperture radar (SAR)). It results from the interference of the coherent and dephased wavelets produced by the backscattering of the interrogating beam due to surface roughness of the order of the beam wavelength or less. In the US context, speckle has its source in the scattering of the ultrasonic pulse from microscopic tissue inhomogeneities.[1] The speckling phenomenon, whose study dates back to the work of Lord Rayleigh (1880), has been thoroughly investigated in the context of laser images[3] using the theory of random walks, and these results have been used to increase our understanding of the speckle generation and statistics in US images. The effects of acoustic speckle on the detectability of low-contrast lesions in US images have been explored both theoretically (by using first and second order statistics as well as statistical decision theory arguments) and experimentally.[4,5] In addition to techniques involving averaging of a number of images of an object produced by spatial or frequency compound scanners,[1,2] aiming at reducing the speckle contrast, several filters acting on a single image have been proposed in the last decade. These filters can be divided into two categories: those which are nonadaptive (or spatially invariant) such as the mean and the median filters, and those whose smoothing properties depend on the image local statistics. The latter (adaptive) filters (e.g., Refs. 6, 7) yield the best results in speckle removal as they satisfy the requirements for maximum noise reduction as well as edge and detail preservation posed by the special nature of the problem. These latter requirements in conjunction with the multiplicative nature of the noise, that we deal with here, make it necessary to resort to nonlinear techniques.[8]

An important class of adaptive speckle filters is what we call here "segmentation-based filters", that is, filtering processes combining segmentation and (nonadaptive) filters. The underlying idea is that, with the aid of a suitable segmentation algorithm, a statistically nonstationary image can be divided into approximately stationary regions which can in turn be processed by filters designed on the basis of the corresponding statistics. Thus, we have a set of filters with each of them corresponding to and operating on a different region of the image with the various regions dictated by the segmentation result. The idea of transforming an image into a set of statistically stationary subimages and feeding these to (nonadaptive) processes is not new. It has been applied to image restoration and compression.[9]

Several approaches to the segmentation of speckle images have been reported (e.g., Ref. 10). A recently introduced segmentation technique, that we have adopted in this work, employs a modification of a well known self-organizing neural network, the Learning Vector Quantizer (LVQ), based on the L_2 mean which is well suited to US images.[11] In Ref. 11, Kotropoulos et al. study the design of the so-called signal-adaptive maximum likelihood filters for the multiplicative and signal-dependent noise cases and apply these filters to the image regions yielded by the L_2 LVQ segmentation with the filters' weighting factor computed as a convex combination of a local signal-to-noise ratio (SNR) measure and a local texture indicator given by the segmentation result. In this paper, we follow a similar approach in that we employ L_2 LVQ to segment the image. In the filtering stage though, we use nonadaptive minimum mean-squared error (MMSE) L-filters designed with the ordering statistical information acquired from the segmentation stage. The proposed filters are tested on a B-mode image of a tissue simulating phantom and the results compare favorably to those produced by a single L-filter designed with the sample statistics of the image considering this as statistically homogeneous. The performance of the various filters are compared on the basis of the resulting receiver operating characteristics (ROC's) and an SNR quantity measuring the dispersion of the image pixels in the lesion and background regions from the corresponding true means. Finally, we present the results of subtracting the SNR image of the original scan from the filtered ones, an operation that improves the lesion detectability.

The paper is organized as follows. In Section 2, we briefly present the LVQ algorithm and its L_2 mean based modification. Section 3 deals with the design of MMSE L-filters for the case of multiplicative noise and known constant signal. Both the unconstrained and unbiased solutions are given. Experimental results from the application of the proposed filters to a simulated B-scan are included in Section 4. Some implementation issues are discussed in Section 5, which concludes the paper.

2. THE LEARNING VECTOR QUANTIZER AND ITS L_2 MEAN BASED MODIFICATION

Learning Vector Quantizer (LVQ)[12,13] is a self-organizing neural network (NN) that belongs to the so-called competitive NN's. It implements a nearest-neighbor classifier using an error correction encoding procedure that could be characterized as a stochastic approximation version of K-means clustering[14]. In this section, we briefly present the LVQ algorithm and its modification based on the L_2 mean (L_2 LVQ) along with a discussion of the reasons that make L_2 LVQ suitable for our application.

Let us first present the basic idea. As in the Vector Quantization (VQ) problem,[14] we have a finite set of variable reference vectors (or "codevectors" in the VQ terminology) $\{\mathbf{w}_i(t); \mathbf{w}_i \in \Re^N, \, i = 1, 2, \ldots, p\}$ and a set of training vectors $\mathbf{x}(t) \in \Re^N$ where t denotes time and we wish to classify the training vectors into p groups represented by the vectors $\mathbf{w}_i$. These representative vectors are obtained by following an iterative procedure where at each time step t the current feature vector $\mathbf{x}(t)$ is compared to all the $\mathbf{w}_i(t)$ and the best-matching $\mathbf{w}_i(t)$ is updated to better comply with $\mathbf{x}(t)$. In this way, in the long run, the different reference vectors tend to become specifically tuned to different domains of the input $\mathbf{x}$. The learning stage of the algorithm is described in the following 4-step procedure:

1. Initialize randomly the reference vectors $\mathbf{w}_i(0)$, $i = 1, 2, \ldots, p$.

2. At time step t, find the "winner" class c such that:

$$||\mathbf{x}(t) - \mathbf{w}_c(t)|| = \min_i \{||\mathbf{x}(t) - \mathbf{w}_i(t)||\}. \tag{1}$$

3. Update the winner:

$$\mathbf{w}_c(t+1) = \mathbf{w}_c(t) + \alpha(t)(\mathbf{x}(t) - \mathbf{w}_c(t)). \tag{2}$$

4. Repeat steps 2 and 3 until convergence.

The gain factor $\alpha(t)$ is a scalar parameter ($0 < \alpha < 1$) which should be a decreasing function of time in order to guarantee the convergence to a unique limit. In the recall procedure, the class with which the input vector $\mathbf{x}(t)$ is most closely associated is determined as in (1) where now $\mathbf{w}_i$ is the i-th reference vector after the convergence of the learning procedure. To be precise we should note that the algorithm described above is the "single-winner" version of LVQ. In its general "multiple-winner" form, step 3 above involves updating not only the winner vector but its neighbors as well with the neighborhood defined either in a topological[12,13] or in a vectorial distance[15] sense. This general form of LVQ has been applied with success to the segmentation of MR images.[16]

It is easy to see that eq. (2) above is in fact a recursive way of computing the average of the training vectors classified to the class c (this is easily verified by choosing $\alpha(t) = 1/(t+1)$) so after the end of the learning phase the reference vectors will correspond to the centroids of the associated classes. It has been shown both analytically and by simulations and experimental B-scans that in the so-called case of "fully-developed speckle", that is, when the density of the scatterers within the insonified medium is large enough, the second and higher order speckle statistics are independent of the tissue characteristics.[17] In other words, the mean echo amplitude level sufficiently characterizes the statistical content of the US image as far as the detection of lesion against the background is concerned. These remarks explain well enough why LVQ, despite its simplicity, is sufficient for a US image segmentation task.

However, it should be noted that the arithmetic mean approximated by the basic LVQ we described so far is not the best possible estimator of the mean level in a US image. It has been proven[18] that the maximum likelihood estimator of the original noiseless image is the L_2 mean of the noisy observations in the case of a multiplicative noise that is of interest here. This result leads us to consider a modification of the standard LVQ algorithm in which the reference vectors correspond to the L_2 mean instead of the arithmetic mean. The learning and recall parts of the modified algorithm which we call L_2 LVQ are exactly analogous to those of the standard LVQ except that the elements of the reference and input vectors are replaced by their squares. This simple modification allows for the computation of the L_2 means providing us at the same time with an algorithm that is proven to be convergent in the mean and in the mean square sense.[11]

3. MMSE L-FILTER DESIGN FOR A KNOWN CONSTANT SIGNAL CORRUPTED BY MULTIPLICATIVE NOISE

The L-filter,[19] defined as a linear combination of the input order statistics, has some distinct advantages making it a right choice for such tasks: it can cope with nonlinear models, it has a relatively simple MMSE design, and furthermore it performs at least as well as, for example, the mean and the median filters as it includes these filters as special cases.[19]

Our model of signal degradation is described by the equation

$$x = sn \tag{3}$$

where x denotes the noisy input observation, s is constant and known, and n is a noise factor with known probability distribution. It is well-known that if the number of scatterers in the tissue examination is sufficiently large, the observed envelope-detected signal x can be considered as a Rayleigh random variable[3–5]. However, we shall use the histogram of grey values as an estimate of the parent distribution of the noisy observations x in each image region in order to cope with the nonstationary nature of ultrasonic images. The output of the L-filter of length M is given by:

$$y = \mathbf{a}^T \mathbf{x} \tag{4}$$

where $\mathbf{a} = (a_1, a_2, \ldots, a_M)^T$ is the L-filter coefficient vector and $\mathbf{x} = (x_{(1)}, x_{(2)}, \ldots, x_{(M)})^T$ is the vector of the observations arranged in ascending order of magnitude (i.e., order statistics). We will design the optimum in the mean-squared error (MSE) sense L-filter for the model (3), that is, determine the vector $\mathbf{a}$ minimizing $E\{(s-y)^2\}$. By using (3) and (4) we obtain:

$$E\{(s-y)^2\} = E\{s^2 + \mathbf{a}^T \mathbf{x}\mathbf{x}^T \mathbf{a} - 2s\mathbf{a}^T \mathbf{x}\} \tag{5}$$

or

$$E\{(s-y)^2\} = s^2 + \mathbf{a}^T \mathbf{R}\mathbf{a} - 2s\mathbf{a}^T \boldsymbol{\mu} \tag{6}$$

where $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^T\}$ is the autocorrelation matrix of the vector of the ordered observations and $\boldsymbol{\mu} = E\{\mathbf{x}\} = (E\{x_{(1)}\}, E\{x_{(2)}\}, \ldots, E\{x_{(M)}\})^T$ is the vector of the expected values of these observations. Setting the derivative of (6) with respect to $\mathbf{a}$ equal to zero yields the following expression for the optimum coefficient vector:

$$\mathbf{a} = s\mathbf{R}^{-1}\boldsymbol{\mu} \tag{7}$$

It remains to compute the ordering statistics $\boldsymbol{\mu}$ and $\mathbf{R}$. Expressions for the evaluation of these quantities are given in Ref. 19 and involve the calculation of the marginal and bivariate probability density functions (pdf's) of the ordered input given its parent distribution:

$$E\{x_{(i)}x_{(j)}\} = \iint xy f_{x_{(i)}x_{(j)}}(x,y)dxdy \quad (i < j) \tag{8}$$

$$E\{x_{(i)}\} = \int x f_{x_{(i)}}(x)dx \tag{9}$$

where

$$f_{x_{(i)}}(x) = K_i F_x^{i-1}(x)[1 - F_x(x)]^{M-i} f_x(x) \tag{10}$$

$$f_{x_{(i)}x_{(j)}}(x,y) = K_{i,j} F_x^{i-1}(x)[F_x(y) - F_x(x)]^{j-i-1} \times [1 - F_x(y)]^{M-j} f_x(x) f_x(y) \tag{11}$$

and

$$K_i = \frac{M!}{(i-1)!(M-i)!} \tag{12}$$

$$K_{i,j} = \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \tag{13}$$

The minimization of the MSE subject to the restriction that $\mathbf{a}$ is unbiased, that is,

$$s = E\{y\} = \mathbf{a}^T\boldsymbol{\mu}, \tag{14}$$

is performed as in the case of additive noise[19] yielding the expression

$$\mathbf{a} = \frac{s\mathbf{R}^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}^T\mathbf{R}^{-1}\boldsymbol{\mu}} \tag{15}$$

for the coefficient vector of the unbiased L-filter.

4. EXPERIMENTAL RESULTS

We have applied our algorithm to a simulated US B-mode image, shown in Figure 1a. It is the B-scan of a tissue mimicking phantom of size 4 cm$\times$4 cm with a circular (disk) lesion in the middle of diameter 2 cm.[20] The density of scatterers is 5000 cm^{-3} and the echo amplitude in the lesion area is 5 dB higher relatively to that of the background. The image has dimensions 241×241 and a resolution of 6 bits/pixel. However, for better visualization, the images presented here were expanded to the range 0 to 255 before being displayed. Because of the high scatterer density, the image of Figure 1a is an example of fully developed speckle. The histograms of the lesion and background regions are plotted in Figure 1b. It is seen that the histogram of grey levels in the background is quite similar to the Rayleigh probability density function, while the corresponding histogram in the lesion resembles the sub-Rayleigh distribution.

Figure 2 shows the result of the segmentation of the original image. Two image regions having different L_2 means have been determined. The dark pixels correspond to the background, while the bright ones to the lesion area. For the classification of the image pixels into two groups, we have employed the L_2 LVQ algorithm with

(a) (b)

Figure 1: (a) US B-scan of a tissue simulating phantom. (b) Histograms of the lesion and the background areas in (a).

parameters $p = 2$ and $N = 49$, trained on a large set of pattern vectors that have been produced by a raster scanning of the image with a 7×7 window.

The histograms of the two regions of the original image classified as dark and bright in Figure 2 have been used as estimates of the parent background and lesion pdf's, i.e., of the pdf of the random variable x in the background and in the lesion areas, respectively, for the design of the associated L-filters following the procedure described in Section 3. Filters of order 3×3 were designed by calculating the ordered statistics from eqs. (10)–(13) and feeding the results to eqs. (8), (9) to estimate the quantities $\mathbf{R}$ and $\boldsymbol{\mu}$ needed in the computation of the filter coefficients (7). The integrals in (8) and (9) were replaced by sums over the range 0 to 63 since the image's grey levels lie in this interval.

Figure 2: Result of the segmentation of Fig. 1a with the modified LVQ NN.

Three pairs of L-filters have been designed by substituting s in (7) with the arithmetic, L_2, and median (5th component of $\boldsymbol{\mu}$) means of the two regions resulted by the segmentation procedure described above. The result

(a) (b)

Figure 3: (a) Output of the single L_2-mean based L-filter. (b) Output of the L_2-mean based L-filter pair.

of the L_2 mean based filter pair is shown in Figure 3b. For comparison purposes, Figure 3a shows the result of a single 3×3 L-filter designed in the same way, but by viewing the original image as stationary. It is seen that the speckle contrast, particularly in the background area, has been reduced, yielding a better discrimination of the lesion against the background. The superiority of the proposed technique in suppressing efficiently the speckle has also been verified in the output of the L-filters designed using the arithmetic or the median mean as an estimate for s respectively. The corresponding filters' coefficients are given in Figure 4.

Figure 4: L_2-mean based L-filter coefficients.

Some detection theoretic performance measures, namely, the probabilities of detection and false alarm, and the receiver operating characteristic[21] have also been used in our comparisons of the filters considered, to allow for numerically comparing their relative performance. The probability of detection (P_D) corresponds to the percentage of pixels of the image in the lesion area that have been correctly classified. The probability of false alarm (P_F) corresponds to the percentage of pixels belonging to the background of the image that were erroneously classified as belonging to the lesion. The probability of detection corresponding to a threshold chosen so that the probability of false alarm is approximately equal to 10% has been tabulated in Table 1 for the original image and its processed versions (linear interpolation was used, where necessary, to estimate P_D from its two closest values). The P_D values of Table 1 verify the enhanced detectability obtained with the filters that exploit the segmentation information compared to their counterparts that are designed with the stationarity assumption. The low probabilities of detection for the median and the mean filters show their inadequacy for this kind of application.

Table 1: Figures of merit for lesion detection on the simulated US B-mode image of Figure 1a.

Method	P_F %	P_D %	Threshold	$\hat{P}_D$ %	Area under ROC
Image Thresholding	8.198 10.128	35.04 38.19	22 21	37.981	0.717116
Median	8.737 10.60	37.45 41.04	21 20	39.8838	0.743840
Arithmetic Mean	8.21 10.37	38.99 43.52	20 19	42.744	0.762216
Single L-filter	9.68 12.0425	41.102 46.1688	19 18	41.7883	0.753955
Segmentation-based L-filters	7.838 10.406	40.3667 45.3982	18 17	44.6027	0.764672

Due to its strong dependence upon the operating point of the detector, the probability of detection, P_D, for a fixed probability of false alarm, P_F, may be proven an inadequate measure of detection performance. A more reliable figure of merit can be derived by examining the receiver operating characteristic (ROC) which is defined as the graph of P_D versus P_F and has been extensively used in evaluating systems for medical diagnosis[22] because of its unique ability for providing a measure of accuracy that is largely independent of decision biases. A single number that can completely characterize the whole ROC is the area under this curve and is included in Table 1. The comparison with this figure of merit is again seen to be favorable for our method. The ROC curves for the images of Figure 3 are shown in Figure 5.

Figure 5: Receiver operating characteristics for the images of Fig. 3.

We have also compared the various filtering strategies from the viewpoint of the dispersion of the background and lesion pixels from the corresponding true sample means relatively to the dispersion in the original image. A

Table 2: Relative dispersion resulting from processing the US B-mode image of Figure 1a.

Method	SNR (dB)
Median	0.903335
Arithmetic Mean	1.51083
Single L-filter	1.63614
Segmentation-based L-filter	2.22718

measure of this relative dispersion, that could be called a signal-to-noise ratio, is defined as

$$\mathrm{SNR} = \frac{\sum_{\text{lesion}}(x_i - m_L)^2 + \sum_{\text{background}}(x_i - m_B)^2}{\sum_{\text{lesion}}(\hat{x}_i - m_L)^2 + \sum_{\text{background}}(\hat{x}_i - m_B)^2} \tag{16}$$

where x_i and $\hat{x}_i$ denote the values of the original and the filtered image, respectively, and m_L, m_B correspond to the average levels in the lesion and background that are estimated from the original image on the basis of our a-priori knowledge of the lesion position, shape, and dimensions. Table 2 summarizes the SNR values (in decibels) for our set of processed images.

A way of increasing the lesion contrast relative to the background is to subtract from an image its associated SNR image.[20] The SNR image is produced by a grey-level encoding of the point SNR evaluated at the corresponding pixels of the original image. The point SNR at pixel P is defined as the ratio of the sample mean to the standard deviation from this mean over a window $\mathcal{W}$ centered at that pixel.[20] That is,

$$\mathrm{SNR}_P = \frac{\mu_P}{\sigma_P} \tag{17}$$

where

$$\mu_P = \frac{1}{N(\mathcal{W})} \sum_{i \in \mathcal{W}} x_i \tag{18}$$

and

$$\sigma_P = \left\{ \frac{1}{N(\mathcal{W}) - 1} \sum_{i \in \mathcal{W}} (x_i - \mu_P)^2 \right\}^{1/2}, \tag{19}$$

with $N(\mathcal{W})$ being the number of pixels enclosed in $\mathcal{W}$. The SNR image of the original image, with $\mathcal{W}$ being a 15×3 window, is shown in Figure 9a. The lesion is recognized as a dark disk-shaped region in this figure. The results of subtracting the SNR image from the L_2 mean based L-filtered images are shown in Figures 9b and c. The image in Figure 9c appears to permit a more accurate discrimination between the target and the background compared to Figure 9b.

Before concluding this section we wish to make some remarks on the L_2 LVQ learning and filters' behavior in the conducted experiments. L_2 LVQ needed at least 5 passes through the image to converge to the result depicted in Figure 2. Longer training periods could not give us a better classifier nor training of the NN with $p > 2$ classes (only two of them were populated at the end). It seems, therefore, that the segmentation in Figure 2 is the best we can obtain for this particular case. It has been found that the unbiased L-filter gives almost identical results with the unconstrained one.

5. CONCLUSIONS

We have presented a method for the suppression of speckle noise in ultrasonic B-mode images based on the idea of segmenting an image into stationary subimages prior to processing each of them with a filter that is designed to be optimum for it on the basis of the (statistical) information provided by the segmentation. Our method employs

(a)

(b)

(c)

Figure 6: (a) SNR image of Fig. 1. (b) Result of subtracting the image of (a) from that of Fig. 3a. (c) Result of subtracting the image of (a) from that of Fig. 3b.

a modification of the LVQ algorithm based on the L_2 mean as a means of segmentation, while its filtering stage uses L-filters whose design is based on a multiplicative noise assumption. Simulation experiments on an image of a tissue mimicking phantom verified the superiority of our approach to the single L-filter and the mean and median cases.

The L_2 LVQ training and the subsequent computation of more than one filters increase the computational complexity of our method to more than twice the computation needed in a conventional approach. However, this need not to be a problem since in medical applications the processing can be done off-line as pointed out in Ref. 7. Moreover, the generalization capability of the LVQ NN, which has already been verified by simulations on SAR images,[23] could be exploited to considerably reduce the computational load by using a single fixed network for segmentation, trained on a sufficiently large and representative sample of images.

In conclusion, we may say that the concept of “segmentation-based filtering,” applied in the problem of speckle noise removal, provides us with methods of increased complexity but of high quality results since the filters can be matched in a natural manner to the local behavior of the image.

6. REFERENCES

1. A. Macovski, *Medical Imaging Systems,* Prentice-Hall, Englewoods Cliffs, N.J., 1983.

2. J. F. Havlice and J. C. Taenzer, "Medical Ultrasonic Imaging: An Overview of Principles and Instrumentation," *Proceedings of the IEEE,* vol. 67, pp. 620–641, Apr. 1979.

3. J. W. Goodman, "Some Fundamental Properties of Speckle," *Journal of the Optical Society of America,* vol. 66, no. 11, pp. 1145–1150, Nov. 1976.

4. S. W. Smith and H. Lopez, "A Contrast-Detail Analysis of Diagnostic Ultrasound Imaging," *Medical Physics,* vol. 9, no. 1, pp. 4–12, Jan/Feb 1982.

5. S. W. Smith, R. F. Wagner, J. M. Sandrik, and H. Lopez, "Low Contrast Detectability and Contrast/Detail Analysis in Medical Ultrasound," *IEEE Transactions on Sonics and Ultrasonics,* vol. SU-30, no. 3, pp. 156–163, May 1983.

6. J.-S. Lee, "A Simple Speckle Smoothing Algorithm for Synthetic Aperture Radar Images," *IEEE Transactions on Systems, Man, and Cybernetics,* vol. SMC-13, no. 1, pp. 85–89, Jan/Feb 1983.

7. T. Loupas, W. N. McDicken, and P. L. Allan, "An Adaptive Weighted Median Filter for Speckle Suppression in Medical Ultrasonic Images," *IEEE Transactions on Circuits and Systems,* vol. CAS-36, no. 1, pp. 129–135, Jan. 1989.

8. I. Pitas and A. N. Venetsanopoulos, *Nonlinear Digital Filters: Principles and Applications,* Kluwer Academic Publishers, Hingham MA, 1990.

9. R. N. Strickland, "Transforming Images into Block Stationary Behavior," *Applied Optics,* vol. 22, no. 10, pp. 1462–1473, 15 May 1983.

10. J.-S. Lee and I. Jurkevich, "Segmentation of SAR Images," *IEEE Transactions on Geoscience and Remote Sensing,* vol. GE-27, no. 6, pp. 674–680, Nov. 1989.

11. C. Kotropoulos, X. Magnisalis, I. Pitas, and M. G. Strintzis, "Nonlinear Ultrasonic Image Processing based on Signal-Adaptive Filters and Self-Organizing Neural Networks," *IEEE Transactions on Image Processing,* January 1994 (to appear).

12. T. Kohonen, *Self-Organization and Associative Memory,* 2nd ed., Springer-Verlang, 1984.

13. T. Kohonen, "The Self-Organizing Map," *Proceedings of the IEEE,* vol. 78, no. 9, pp. 1464–1480, Sept. 1990.

14. Y. Linde, A. Buzo, and R. M. Gray, "An Algorithm for Vector Quantizer Design," *IEEE Transactions on Communications,* vol. COM-28, no. 1, pp. 84–95, Jan. 1980.

15. J. A. Kangas, T. K. Kohonen, and J. T. Laaksonen, "Variants of Self-Organizing Maps," *IEEE Transactions on Neural Networks,* vol. 1, no. 1, pp. 93–99, Mar. 1990.

16. A. Springub, D. Scheppelmann, and H.-P. Meinzer, "Segmentation of Multisignal Images with Kohonen's Self-Learning Topological Map," *Proc. Conf. Computer Analysis of Images and Patterns,* pp. 148–152, Dresden, Germany, Sept. 1991.

17. B. J. Oosterveld, J. M. Thijssen, and W. A. Verhoef, "Texture of B-Mode Echograms: 3-D Simulations and Experiments of the Effects of Diffraction and Scatterer Density," *Ultrasonic Imaging,* vol. 7, pp. 142–160, 1985.

18. C. Kotropoulos and I. Pitas, "Optimum Nonlinear Signal Detection and Estimation in the Presence of Ultrasonic Speckle," *Ultrasonic Imaging,* vol. 14, pp. 249–275, 1992.

19. A. C. Bovik, T. S. Huang, and D. C. Munson, "A Generalization of Median Filtering Using Linear Combinations of Order Statistics," *IEEE Transactions on Acoustics, Speech, and Signal Processing,* vol. ASSP-31, no. 6, pp. 1342–1350, Dec. 1983.

20. J. T. M. Verhoeven, J. M. Thijssen, and A. G. M. Theeuwes, "Lesion Detection by Echographic Image Processing: Signal-To-Noise Ratio Imaging," *Ultrasonic Imaging,* vol. 13, pp. 238–251, 1991.

21. H. L. VanTrees, *Detection, Estimation, and Modulation Theory,* John Wiley & Sons, 1968.

22. J. A. Swets et al., "Assessment of Diagnostic Technologies," *Science,* vol. 205, no. 4407, pp. 753–759, 24 Aug. 1979.

23. S. P. Luttrel, "Image Compression Using a Multilayer Neural Network," *Pattern Recognition Letters,* vol. 10, pp. 1–7, July 1989.