# Application of Neural Networks and Order Statistics Filters to Speckle Noise Reduction in Remote Sensing Imaging

E. Kofidis[1], S. Theodoridis[2], C. Kotropoulos[3], and I. Pitas[3]

[1] University of Patras, Department of Computer Engineering, 265 00 Patras, Greece.

[2] University of Athens, Department of Informatics, 157 71 Athens, Greece. e-mail: stheodor@di.uoa.gr

[3] University of Thessaloniki, Department of Informatics, 540 06 Thessaloniki, Greece.

**Summary.** A novel approach to suppression of speckle noise in remote sensing imaging based on a combination of segmentation and optimum L-filtering is presented. With the aid of a suitable modification of the Learning Vector Quantizer (LVQ) neural network, the image is segmented in regions of (approximately) homogeneous statistics. For each of the regions a minimum mean-squared-error (MMSE) L-filter is designed, by using the histogram of grey levels as an estimate of the parent distribution of the noisy observations and a suitable estimate of the (assumed constant) original signal in the corresponding region. Thus, a bank of L-filters results, with each of them corresponding to and operating on a different image region. Simulation results are presented, which verify the (qualitative and quantitative) superiority of our technique over a number of commonly used speckle filters.

## 1. Introduction

One of the major problems encountered in Remote Sensing and Ultrasonic Imaging is speckle noise reduction. This type of noise contamination, which is met in all coherent imaging systems, results from the scattering of the transmitted wave from terrain inhomogeneities which are small with respect to the wavelength [4]. A multiplicative model for speckle noise is implied by the fact that the standard deviation is directly proportional to the mean and it has been verified experimentally [9].[1]

The speckle artifact severely degrades the information content of an image and poses difficulties in the image analysis phase. Thus it is desirable to suppress the noise while at the same time retaining the useful information unimpaired. Several algorithms have been proposed aiming at reducing speckle noise in images (e.g., [2, 9, 12, 14, 11]). Since the ultimate goal of any speckle suppression scheme should be the reduction of speckle contrast to enhance the information content of the image, edge and detail preservation are

[1] Nevertheless, it must be mentioned that speckle noise is only approximately multiplicative in regions of the object containing fine details that cannot be resolved by the imaging system [18] and the experimental verification in [9] was based only on flat areas of the image. In spite of this, speckle noise is usually modelled as multiplicative in practice.

crucial in a speckle filter along with noise reduction. Thus, spatially-varying filters are required that are also able to deal with the *nonlinear* model governing the degradation process [16].

An important class of adaptive filters is what we call here "segmentation-based filters", that is, filtering processes combining segmentation and (non-adaptive) filters. The underlying idea is that, with the aid of a suitable segmentation algorithm, a statistically non-stationary image can be divided into approximately stationary regions which can, in turn, be processed by filters designed on the basis of the corresponding statistics. Thus, we have a set of filters with each of them corresponding to and operating on a different region of the image, with the various regions being dictated by the segmentation result. In this paper, we report such an approach to speckle suppression employing a modification of the Learning Vector Quantizer neural network at the segmentation stage and non-adaptive minimum mean-squared error (MMSE) L-filters at the filtering stage, designed with the ordering statistical information acquired from the segmentation stage. The proposed filters have been tested on a simulated image containing a bright target in a dark background and the results compare favourably to those produced by a single L-filter designed with the sample statistics of the image considering this as statistically homogeneous. Results of comparison with a number of commonly used speckle filters are also given, which rank our method among the first positions. The noise-smoothing performances of the various filters are compared on the basis of the resulting receiver operating characteristics (ROC's) and an SNR quantity measuring the dispersion of the image pixels in the target and background regions from the corresponding true means. The contrast enhancement effect of the filters is quantitatively assessed through a target contrast measure.

The paper is organised as follows. Our method is presented in detail in section 2. Experimental results are included in section 3, along with a comparison with a number of other well known filtering strategies. Some implementation issues are discussed in section 4, which concludes the paper.

## 2. Segmentation-Based L-Filtering

In this section we present an adaptive nonlinear approach to speckle suppression in images. The adaptivity of our method comes from the fact that the image is first segmented into regions of different characteristics and each of the resulting regions is processed by a different filter. L-filters are employed to deal with the nonlinear nature of the noise. A number of approaches to the segmentation of speckle images have been reported (e.g., [10]). A recently introduced segmentation technique, that we have adopted in this work, employs a modification of a well known self-organising neural network, the Learning Vector Quantizer (LVQ), based on the $L_2$ mean which has been shown to be more suitable for speckle images [7].

In the sequel, a brief presentation of LVQ is given followed by the description of its modification, $L_2$ LVQ, along with a discussion of the need for this modified form. The derivation of the MMSE L-filter for the case of a known constant signal corrupted by noise is included both in the unconstrained and constrained (unbiased) cases. In most cases it is unrealistic to assume that the signal is constant. However, since the filters are matched to specific regions of the image, this simplifying assumption is a good approximation of the reality for practical purposes.

## 2.1 The Learning Vector Quantizer and its $L_2$ Mean Based Modification

Learning Vector Quantizer (LVQ) [6] is a self-organising neural network (NN) that belongs to the so-called competitive NN's. It implements a nearest-neighbour classifier using an error correction encoding procedure that could be characterised as a stochastic approximation version of $K$-means clustering. Let us first present the basic idea. As in the Vector Quantization (VQ) problem, we have a finite set of variable reference vectors (or "code vectors" in the VQ terminology) $\{\mathbf{w}_i(t); \mathbf{w}_i \in \mathcal{R}^N, i = 1, 2, \ldots, p\}$ and a set of training vectors $\mathbf{x}(t) \in \mathcal{R}^N$ where $t$ denotes time and we wish to classify the training vectors into $p$ classes represented by the vectors $\mathbf{w}_i$. These representative vectors are obtained by following an iterative procedure where at each iteration step $t$ the current feature vector $\mathbf{x}(t)$ is compared to all the $\mathbf{w}_i(t)$ and the best-matching $\mathbf{w}_i(t)$ is updated to better comply with $\mathbf{x}(t)$. In this way, in the long run, the different reference vectors tend to become specifically tuned to different domains of the input $\mathbf{x}$. The learning stage of the algorithm is described in the following 4-step procedure:

i. Initialise randomly the reference vectors $\mathbf{w}_i(0)$, $i = 1, 2, \ldots, p$.
ii. At time step $t$, find the "winner" class $c$ such that:

$$\|\mathbf{x}(t) - \mathbf{w}_c(t)\| = \min_i \{\|\mathbf{x}(t) - \mathbf{w}_i(t)\|\} . \tag{2.1}$$

iii. Update the winner:

$$\mathbf{w}_c(t+1) = \mathbf{w}_c(t) + \alpha(t)(\mathbf{x}(t) - \mathbf{w}_c(t)). \tag{2.2}$$

iv. Repeat steps (ii) and (iii) until convergence.

The gain factor $\alpha(t)$ is a scalar parameter ($0 < \alpha < 1$) which should be a decreasing function of time in order to guarantee the convergence to a unique limit. In the recall procedure, the class with which the input vector $\mathbf{x}(t)$ is most closely associated is determined as in (2.1) where now $\mathbf{w}_i$ is the $i$-th reference vector after the convergence of the learning procedure.[2]

[2] To be precise, we should note that the algorithm described above is the "single-winner" version of LVQ. In its general "multiple-winner" form, step iii above

It is easy to see that eq. (2.2) above is in fact a recursive way of computing the average of the training vectors classified to the class $c$ (this is easily verified by choosing $\alpha(t) = 1/(t+1)$). Thus, after the end of the learning phase, the reference vectors will correspond to the centroids of the associated classes. However, it should be noted that the arithmetic mean approximated by the basic LVQ, described so far, is not the best possible estimator of the mean level in a speckle image. It has been proved [8] that the maximum likelihood estimator of the original noiseless image is the $L_2$ mean [16] (scaled by $\frac{\sqrt{\pi}}{2}$) of the noisy observations. This result leads us to consider a modification of the standard LVQ algorithm, in which the reference vectors correspond to the $L_2$ mean instead of the arithmetic mean. The learning and recall parts of the modified algorithm, which we call $L_2$ LVQ, are exactly analogous to those of the standard LVQ except that the elements of the reference and input vectors are replaced by their squares. This simple modification allows for the computation of the $L_2$ means providing us at the same time with an algorithm that is proven to be convergent in the mean and in the mean square sense [7].

## 2.2 MMSE L-Filter Design for a Known Constant Signal Embedded in Noise

The L-filter [1], defined as a linear combination of the input order statistics, has some distinct advantages, making it a right choice for tasks such as the one treated here: it can cope with nonlinear models, it has a relatively simple MMSE design, and furthermore it performs at least as well as, for example, the mean and the median filters, as it includes these filters as special cases [1].

In the sequel, $s$ denotes the constant and known signal, which is corrupted by white[3] noise, independent of $s$, yielding the noisy observation $x$. The output of the L-filter of length $M$ is given by:

$$y = \mathbf{a}^T \mathbf{x} \tag{2.3}$$

where $\mathbf{a} = (a_1, a_2, \ldots, a_M)^T$ is the L-filter coefficient vector and $\mathbf{x} = (x_{(1)}, x_{(2)}, \ldots, x_{(M)})^T$ is the vector of the observations arranged in ascending order of magnitude (i.e., order statistics). We will design the optimum in the mean-squared error (MSE) sense L-filter, that is, determine the vector $\mathbf{a}$ minimising $E\{(s-y)^2\}$. By using (2.3) we obtain:

involves updating not only the winner vector but its neighbours as well with the neighbourhood defined either in a topological [6] or in a vectorial distance [5] sense.

[3] In fact, speckle noise is locally correlated. Smith et al. [17] argue that, for the observations to be independent, they must belong to different speckle correlation cells. Since our purpose is to apply filters scanning the image in raster fashion, such a recommendation cannot be used directly, thus the whiteness assumption is made to approximate the real situation.

$$E\{(s-y)^2\} = s^2 + \mathbf{a}^T\mathbf{R}\mathbf{a} - 2s\mathbf{a}^T\boldsymbol{\mu} \tag{2.4}$$

where $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^T\}$ is the autocorrelation matrix of the vector of the ordered observations and $\boldsymbol{\mu} = E\{\mathbf{x}\} = (E\{x_{(1)}\}, E\{x_{(2)}\}, \ldots, E\{x_{(M)}\})^T$ is the vector of the expected values of these observations. Setting the derivative of (2.4) with respect to $\mathbf{a}$ equal to zero yields the following expression for the optimum coefficient vector:

$$\mathbf{a} = s\mathbf{R}^{-1}\boldsymbol{\mu} \tag{2.5}$$

It remains to compute the ordering statistics $\boldsymbol{\mu}$ and $\mathbf{R}$. Expressions for the evaluation of these quantities are given in [1] and involve the calculation of the marginal and bivariate probability density functions (pdf's) of the ordered input given its parent distribution:

$$E\{x_{(i)}x_{(j)}\} = \iint xy f_{x_{(i)}x_{(j)}}(x,y)dxdy \quad (i<j) \tag{2.6}$$

$$E\{x_{(i)}\} = \int x f_{x_{(i)}}(x)dx \tag{2.7}$$

where

$$f_{x_{(i)}}(x) = K_i F_x^{i-1}(x)[1-F_x(x)]^{M-i} f_x(x) \tag{2.8}$$

$$f_{x_{(i)}x_{(j)}}(x,y) = K_{i,j} F_x^{i-1}(x)[F_x(y)-F_x(x)]^{j-i-1} \times[1-F_x(y)]^{M-j} f_x(x) f_x(y) \tag{2.9}$$

and

$$K_i = \frac{M!}{(i-1)!(M-i)!} \tag{2.10}$$

$$K_{i,j} = \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \tag{2.11}$$

Notice that when we are dealing with digital images, the above random variables are of discrete type. Thus, the integrals in eqs. (2.6), (2.7) are in fact discrete sums.

The minimisation of the MSE subject to the constraint that $\mathbf{a}$ provides an unbiased estimate of $s$, i.e.,

$$s = E\{y\} = \mathbf{a}^T\boldsymbol{\mu}, \tag{2.12}$$

is performed as in the case of additive noise [1] yielding the expression

$$\mathbf{a} = \frac{s\mathbf{R}^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}^T\mathbf{R}^{-1}\boldsymbol{\mu}} \tag{2.13}$$

for the coefficient vector of the unbiased L-filter.

## 3. Experimental Results

To test the performance of our method in speckle smoothing and detail preservation, an image consisting of two regions, the target and the background, has been used. For the classification of the image pixels into two groups, we have employed the $L_2$ LVQ algorithm with parameters $p = 2$ and $N = 49$, trained on a large set of pattern vectors that have been produced by a raster scanning of the image with a $7 \times 7$ window. The histograms of the two regions produced by the segmentation have been used as estimates of the parent background and target pdf's, i.e., of the pdf of the random variable $x$ in the background and in the target areas, respectively, for the design of the associated L-filters. Filters of order $3 \times 3$ were designed by calculating the ordered statistics from eqs. (2.8)–(2.11) and feeding the results to eqs. (2.6), (2.7) to estimate the quantities $\mathbf{R}$ and $\mu$ needed in the computation of the filter coefficients (2.5). The integrals in (2.6) and (2.7) were replaced by sums over the range 0 to 63 since the image's grey levels lie in this interval.

A pair of L-filters have been designed by substituting $s$ in (7) with the $L_2$ means of the two regions resulted by the segmentation procedure described above.

The arithmetic mean and the median filters have also been used in our comparisons along with a number of well-known speckle filters:

i. Homomorphic filter [15]
ii. Frost filter [3, 2]
iii. Sigma filter [9]
iv. Variable-length Median filter [12]
v. Taylor filter [14]

Some detection theoretic performance measures, namely, the probabilities of detection and false alarm, and the receiver operating characteristic have also been used in our comparisons of the filters considered, to allow for numerically comparing their relative performance. The probability of detection corresponding to a threshold chosen so that the probability of false alarm is approximately equal to 10% has been tabulated in table 3.1 for the original image and its processed versions (linear interpolation was used, where necessary, to estimate $P_D$ from its two closest values). The $P_D$ values listed in table 3.1 verify the enhanced detectability obtained by the filters that exploit the segmentation information, compared to their counterparts that are designed with the stationarity assumption. The low probabilities of detection for the median and the mean filters show their inadequacy for this kind of application.

Due to its strong dependency upon the operating point of the detector, the probability of detection $P_D$, for a fixed probability of false alarm $P_F$, may be proved an inadequate measure of detection performance. A more reliable figure of merit can be derived by examining the receiver operating characteristic (ROC). A single number that can completely characterise the

Table 3.1. Detection Performance Measures

| Method | $P_F$ % | $P_D$ % | Threshold | $P_D$ % | Area under ROC |
|---|---|---|---|---|---|
| Image Thresholding | 8.198<br>10.128 | 35.04<br>38.19 | 22<br>21 | 37.981 | 0.717116 |
| Median | 8.737<br>10.60 | 37.45<br>41.04 | 21<br>20 | 39.8838 | 0.743840 |
| Average | 9.175<br>11.48 | 40.907<br>45.62 | 20<br>19 | 42.592 | 0.761905 |
| Homomorphic | 8.0845<br>10.051 | 36.91<br>40.987 | 20<br>19 | 40.88 | 0.754272 |
| Frost | 8.71<br>12.3 | 43.73<br>50.11 | 17<br>16 | 46.021 | 0.77281 |
| Sigma | 9.17<br>11.476 | 40.845<br>45.61 | 20<br>19 | 42.56 | 0.761576 |
| V. L. Median | 8.78<br>10.68 | 37.44<br>40.854 | 21<br>20 | 39.6336 | 0.731703 |
| Taylor | 9.175<br>11.48 | 40.9<br>45.61 | 20<br>19 | 42.592 | 0.761882 |
| L-filter | 8.674<br>10.7 | 38.8<br>43.343 | 21<br>20 | 41.77 | 0.758334 |
| L-filter pair | 7.838<br>10.406 | 40.3667<br>45.3982 | 18<br>17 | 44.6027 | 0.764672 |

whole ROC is the area under this curve and is included in table 3.1. The comparison with respect to this figure of merit is again seen to be favourable for our method.

We have also compared the various filtering strategies from the viewpoint of the dispersion of the background and target pixels from the corresponding true sample means relatively to the dispersion in the original image. A measure of this relative dispersion, that could be called a signal-to-noise ratio, is defined for the target area as

$$\mathrm{SNR}_T = \frac{\sum_{\text{target}}(x_i - m_T)^2}{\sum_{\text{target}}(\hat{x}_i - m_T)^2} \tag{3.1}$$

where $x_i$ and $\hat{x}_i$ denote the values of the original and the filtered image, respectively, and $m_T$ corresponds to the average level in the target that is estimated from the original image on the basis of our a-priori knowledge of the target position, shape, and dimensions. The background SNR, $\mathrm{SNR}_B$, is similarly defined. Table 3.2 summarises the SNR values (in decibels) for our set of processed images.

The results presented thus far, demonstrate that our method outperforms all of the filters considered except for the Frost filter, which attains significantly higher values for the ROC area and the SNR compared to the segmentation-based approach. Nevertheless, these higher figures of merit for the Frost filter are at the cost of lower target contrast and an amount of

**Table 3.2.** Relative dispersion in the target and background areas

| Method | $SNR_B$ (dB) | $SNR_T$ (dB) |
|---|---|---|
| Median | 0.914986 | 0.885867 |
| Arithmetic Mean | 1.55226 | 1.5066 |
| Homomorphic | 1.56488 | 1.50149 |
| Frost | 4.59596 | 4.02855 |
| Sigma | 1.54537 | 1.49875 |
| V. L. Median | 0.812148 | 0.613674 |
| Taylor | 1.55216 | 1.5066 |
| L-filter | 1.09915 | 1.04087 |
| L-filter pair | 2.28008 | 2.1488 |

blurring. This could be expected since, as noted in [11], Frost's filter cannot adequately smooth homogeneous areas and preserve heterogeneous areas at the same time. A quantitative verification of this point is provided by the following measure of target contrast

$$C = \frac{m_T - m_B}{m_T + m_B} \tag{3.2}$$

where, as before, $m_T$ and $m_B$ denote the average levels in the target and the background, respectively. The contrast values for the original as well as the filtered images are tabulated in table 3.3. Note that the Frost filter yields the lowest target contrast among all the filters studied here, with the highest value obtained through our method.

**Table 3.3.** Target contrast

| Method/Image | Contrast |
|---|---|
| Original | 0.230441 |
| Median | 0.230212 |
| Average | 0.230541 |
| Homomorphic | 0.231008 |
| Frost | 0.187298 |
| Sigma | 0.230571 |
| V. L. Median | 0.228513 |
| Taylor | 0.230541 |
| L-filter | 0.238434 |
| L-filter pair | 0.238952 |

## 4. Conclusions

We have presented a method for the suppression of speckle noise in remote-sensing imagery based on the idea of segmenting an image into stationary sub-images prior to processing each of them with a filter that is designed to be optimal for each particular sub-image on the basis of the (statistical) information provided by the segmentation. Our method employs a modification of the LVQ algorithm based on the $L_2$ mean as a means of segmentation, while its filtering stage uses L-filters that are optimal in the MSE sense. The simulation results verified the superiority of our approach to the single L-filter as well as to a number of commonly used speckle filters.

The $L_2$ LVQ training and the subsequent computation of more than one filters increase the computational complexity of our method to more than twice the computation needed in a conventional approach. However, this need not to be a problem if the processing can be done off-line. Moreover, the generalisation capability of the LVQ NN, which has already been verified by simulations on SAR images [13], could be exploited to considerably reduce the computational load by using a single fixed network for segmentation, trained on a sufficiently large and representative sample of images.

## References


1. A. C. Bovik, T. S. Huang, and D. C. Munson, "A Generalisation of Median Filtering Using Linear Combinations of Order Statistics", *IEEE Transactions Acoustics, Speech, and Signal Processing*, vol. 31, no. 6, December 1983, pp. 1342–1350.
2. V. S. Frost, J. A. Stiles, K. S. Shanmugam, and J. C. Holtzman, "A Model For Radar Images and Its Application to Adaptive Digital Filtering of Multiplicative Noise", *IEEE Transactions Pattern Analysis and Machine Intelligence*, vol. 4, no. 2, March 1982, pp. 157–165.
3. V. S. Frost, J. A. Stiles, K. S. Shanmugam, J. C. Holtzman, and S. A. Smith, "An Adaptive Filter for Smoothing Noisy Radar Images", *Proceedings IEEE*, vol. 69, no. 1, Jan. 1981, pp. 133–135.
4. J. W. Goodman, "Some Fundamental Properties of Speckle", *J. Optical Society of America*, vol. 66, no. 11, Nov. 1976, pp. 1145–1150.
5. J. A. Kangas, T. K. Kohonen, and J. T. Laaksonen, "Variants of Self-Organizing Maps", *IEEE Transactions Neural Networks*, vol. 1, no. 1, March 1990, pp. 93–99.
6. T. Kohonen, "The Self-Organizing Map", *Proceedings IEEE*, vol. 78, no. 9, September 1990, pp. 1464–1480.
7. C. Kotropoulos, X. Magnisalis, I. Pitas, and M. G. Strintzis, "Nonlinear Ultrasonic Image Processing based on Signal-Adaptive Filters and Self-Organizing Neural Networks", *IEEE Transactions Image Processing*, vol. 3, no. 1, Jan. 1994, pp. 65–77.
8. C. Kotropoulos and I. Pitas, "Optimum Nonlinear Signal Detection and Estimation in the Presence of Ultrasonic Speckle", *Ultrasonic Imaging*, vol. 14, 1992, pp. 249–275.
9. J.-S. Lee, "A Simple Speckle Smoothing Algorithm for Synthetic Aperture Radar Images", *IEEE Transactions Systems, Man, and Cybernetics*, vol. 13, no. 1, Jan/Feb. 1983, pp. 85–89.
10. J.-S. Lee and I. Jurkevich, "Segmentation of SAR Images", *IEEE Transactions Geoscience and Remote Sensing*, vol. 27, no. 6, November 1989, pp. 674–680.

11. A. Lopes, R. Touzi, and E. Nezry, "Adaptive Speckle Filters and Scene Heterogeneity", *IEEE Transactions Geoscience and Remote Sensing,* vol. 28, no. 6, November 1990, pp. 992–1000.
12. T. Loupas, W. N. McDicken, and P. L. Allan, "An Adaptive Weighted Median Filter for Speckle Suppression in Medical Ultrasonic Images", *IEEE Transactions Circuits and Systems,* vol. 36, no. 1, January 1989, pp. 129–135.
13. S. P. Luttrel, "Image Compression Using a Multilayer Neural Network", *Pattern Recognition Letters,* vol. 10, July 1989, pp. 1–7.
14. C. R. Moloney and M. E. Jernigan, "Nonlinear Adaptive Restoration of Images with Multiplicative Noise", *Proceedings ICASSP '89,* pp. 1433–1436.
15. A. V. Oppenheim, R. W. Schafer, and T. G. Stockham, Jr., "Nonlinear Filtering of Multiplied and Convolved Signals", *Proceedings IEEE,* vol. 56, August 1968, pp. 1264–1291.
16. I. Pitas and A. N. Venetsanopoulos, *Nonlinear Digital Filters: Principles and Applications,* Kluwer Academic Publishers, Hingham MA, 1990.
17. S. W. Smith, R. F. Wagner, J. M. Sandrik, and H. Lopez, "Low Contrast Detectability and Contrast/Detail Analysis in Medical Ultrasound", *IEEE Transactions Sonics and Ultrasonics,* vol. 30, no. 3, May 1983, pp. 164–173.
18. M. Tur, K. C. Chin, and J. W. Goodman, "When is Speckle Noise Multiplicative?", *Applied Optics,* vol. 21, no. 7, April 1982, pp. 1157–1159.