

CELLULAR LMS L -FILTERS FOR NOISE SUPPRESSION IN STILL IMAGES AND IMAGE SEQUENCES

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ABSTRACT

A novel class of nonlinear adaptive L -filters based on cellular neural networks topology is presented. Like cellular neural systems and cellular automata as well, processing nodes, called cells, communicate with each other directly only through its nearest neighbors exchanging information. Each cell is an adaptive LMS L -filter. The proposed filters share the best features of both adaptive filters and cellular neural network topologies; their adaptive structure tracks image nonstationarities and their local interconnection feature makes it suitable for VLSI implementation. Cellular adaptive LMS L -filters are suited for high-speed parallel adaptive image filtering. Some interesting applications to image and image sequence filtering will be demonstrated in this paper.

1. INTRODUCTION

Adaptive signal processing has exhibited a tremendous growth in the past two decades. Adaptive filters have been applied in a wide variety of problems [1]. The most widely known adaptive filters are linear ones. However, such filters are not suitable for applications where the noise is impulsive or where the signal is strongly nonstationary e.g. in image processing. In the later case, a multitude of nonlinear techniques has been proved a successful alternative to linear techniques [2]. One of the best known families is based on the order statistics, i.e., on data ordering. Among the filters that belong to this family are the L -filters [3]. The adaptation of the coefficients employed in order statistic filters by using linear adaptive signal processing techniques, e.g. the Least Mean Squares (LMS) algorithm, or the Recursive Least Squares (RLS) algorithm, has received much attention in the literature [4]. The majority of adaptive order statistic filters has been applied to 1-d signals. However, two-dimensional LMS L -filters have been studied in [5,6]. The major contribution of this paper is in the design of a novel adaptive L -filter structure that is reminiscent of the cellular neural network topology [7,8] in order to cope with the nonstationary nature of images, and moreover, of image sequences. Due to the fact that all the interactions between the processing cells (that are adaptive L -filters themselves) are mainly confined to a finite neighborhood around each processing cell, the novel adaptive L -filter structure is able to converge to a set of optimal L -filters that match the local

image statistics. On the contrary, a single adaptive L -filter is expected to track nonstationarity only in environments of slowly varying statistics.

The design of the novel class of the adaptive LMS L -filters called *Cellular Adaptive LMS L -filters* is presented in Section 2. A stability analysis is presented in Section 3 and the practical question of dynamic range is derived in Section 4. Computer simulation and typical dynamic behaviors of the proposed filter are discussed in Section 5.

2. DESIGN OF CELLULAR ADAPTIVE LMS L -FILTERS

In this section we shall derive the adaptation formula for the Cellular adaptive LMS L -filter in the case of an image corrupted by zero-mean additive noise. Let us assume that we scan the image in a specific order. The observed image pixel value $x(i)$ can be expressed as a sum of an arbitrary true pixel value $d(i)$ plus zero-mean additive white noise, i.e. :

$$x(i) = s(i) + n(i) \quad (1)$$

For notation simplicity, we shall use only one index i to denote the spatial coordinates. Let L denote the number of image rows/columns. Depending on the image scanning method, each pixel (i_1, i_2) ; $i_1, i_2 = 1, \dots, L$ can be represented by a single running index i . In the case of a still image, let $\mathbf{x}(i)$ be the $M \times 1$ tap-input vector formed by the image pixel values that lie inside a $(2\nu + 1) \times (2\nu + 1)$ squared window centered on pixel i , i.e.,

$$\mathbf{x}(i) = [x(i_1 - \nu, i_2 - \nu), \dots, x(i_1, i_2), \dots, x(i_1 + \nu, i_2 + \nu)]^T. \quad (2)$$

Let $x_{(1)}(i) \leq x_{(2)}(i) \leq \dots \leq x_{(M)}(i)$ denote the observed pixel values arranged in ascending order of magnitude that form the vector of the order statistics within the observation window $\mathbf{x}_r(i) = [x_{(1)}(i), x_{(2)}(i), \dots, x_{(M)}(i)]^T$. Then, the output of the L -filter of length M is $y(i) = \mathbf{a}^T(i)\mathbf{x}_r(i)$, where $\mathbf{a}(i)$ is the L -filter coefficient vector. An ordinary LMS adaptive L -filter would evaluate first the estimation error at pixel i $\varepsilon(i) = d(i) - \mathbf{a}^T(i)\mathbf{x}_r(i)$ and it would update the L -filter coefficients as follows [4]-[6]:

$$\mathbf{a}(i+1) = \mathbf{a}(i) + \mu \varepsilon(i) \tilde{\mathbf{x}}_r(i). \quad (3)$$

It is seen that the adaptation of the L -filter coefficients at pixel i depends explicitly on all the past values of the de-

sired signal $d(l)$ and the ordered-tap input vectors $\mathbf{x}_r(l)$, $l < i$. Such a direct dependence on the past history in a nonstationary environment, in addition to the transitions introduced by the scanning method, prevents the filter to converge locally to a meaningful solution, i.e., to a filter that matches the local statistics. This observation motivated us to adopt an approach where the direct interactions between image pixels are limited within a finite local neighborhood. It is well known that cellular neural systems and cellular automata share the same property: all the interactions are local within a finite radius [7,8]. Accordingly, we have borrowed the cellular network topology in order to design a novel adaptive L -filter structure to be called *cellular adaptive L-filter* hereafter.

Let us describe the proposed cellular adaptive L -filter structure. It is a $L \times L$ cellular network having $L \times L$ processing cells, one assigned to each image pixel, arranged in L rows and L columns. Each processing cell is denoted by $C(i)$ and it is an adaptive LMS L -filter having coefficient vector $\mathbf{a}(i)$. Around each processing cell a neighborhood is defined. Let $\mathbf{a}(i; k)$ denote the L -filter coefficients of the i -th processing cell at the k -th iteration. At $k = 0$, all the processing cells are randomly initialized. At each iteration, all the processing cells evaluate synchronously an estimation error of the form:

$$\varepsilon(i; k) = s(i; k) - \mathbf{a}^T(i; k) \mathbf{x}_r(i; k); \quad i = 1, \dots, L^2. \quad (4)$$

Following the approach in [2], the steepest descent recursion expression of the new coefficients as a function of the old coefficients is given by:

$$\mathbf{a}(i; k + 1) = \mathbf{a}(i; k) - \frac{1}{2} \mu \nabla J(i; k) \quad (5)$$

where μ is the step-size and the mean-squared error is given by:

$$J(i; k) = E \left[\left(\frac{1}{N} \sum_{j \in \mathcal{N}(i)} \varepsilon(j; k) \right)^2 \right] \quad (6)$$

where N is the cardinality of the processing cells in $\mathcal{N}(i)$. By differentiating the mean-squared error $J(i; k)$ of (6) with respect to coefficients $\mathbf{a}(i; k)$ we get the gradient of $J(i; k)$ and by using the LMS algorithm, we form the adaptation algorithm. All the adjacent processing cells to pixel i , i.e., all the processing cells inside the r -neighborhood of all $C(i)$ exchange their estimation errors, and $C(i)$ evaluates an averaged estimation error:

$$\bar{\varepsilon}(i; k) = \frac{1}{N} \sum_{j \in \mathcal{N}(i)} \varepsilon(j; k); \quad i = 1, \dots, L^2 \quad (7)$$

This interaction within a neighborhood can be seen in Figure 1. Thus, the L -filter coefficients of the i -th processing cell are updated as follows:

$$\mathbf{a}(i; k + 1) = \mathbf{a}(i; k) + \mu \bar{\varepsilon}(i; k) \mathbf{x}_r(i; k); \quad i = 1, \dots, L^2. \quad (8)$$

which is a cell dynamic equation.

It can easily be understood that in the case of still images $d(i; k) = d(i)$, $\mathbf{x}_r(i; k) = \mathbf{x}_r(i)$ and $s(i; k) = s(i)$, for all iterations $k = 1, 2, \dots$. In the case of image sequences, the index k could correspond either to the frame that is

being filtered or to an iteration index. Unlike the ordinary LMS algorithm ([5],[6]) for adapting the L -filter coefficients, which is a recursive algorithm that cannot be implemented in parallel, the cellular adaptive L -filters can easily be parallelized. Indeed, each processing cell exchanges its estimation error with the adjacent cells and all the remaining operations are local, i.e., they can be performed at each processing cell, and moreover, in parallel.

3. STABILITY

The basic function of a cellular adaptive L -filter is to transform an input image into a corresponding output image. This means that our cellular adaptive L -filter must always converge to a constant steady state following a transient period after initialization. In this section, we shall discuss the convergence property for Cellular Adaptive LMS L -filters. Our system is a discrete autonomous system therefore, in order the system to converge, the eigenvalues of the matrix \mathbf{A} must lie inside the unit circle. First, let us recast the cell dynamic equation (8) in the following form:

$$\mathbf{a}(i; k + 1) = \mathbf{A}(i)^k \mathbf{a}(i; 0) + \sum_{p=0}^{k-1} \mathbf{A}(i)^{k-1-p} (\mathbf{g}(i) - \mathbf{f}(i; p)) + (\mathbf{g}(i) - \mathbf{f}(i; k)) \quad (9)$$

where:

$$\mathbf{A}(i) = \mathbf{I} - \frac{\mu}{N} \mathbf{x}_r(i) \mathbf{x}_r^T(i) \quad (10)$$

$$\mathbf{f}(i; k) = \frac{\mu}{N} \sum_{j \in \mathcal{N}^*(i)} \mathbf{x}_r^T(j) \mathbf{a}(j; k) \mathbf{x}_r(i) \quad (11)$$

$$\mathbf{g}(i) = \frac{\mu}{N} \sum_{j \in \mathcal{N}(i)} s(j) \mathbf{x}_r(i) \quad (12)$$

and $\mathcal{N}^*(i)$ is the neighborhood of pixel i excluding pixel i . If we denote by $\mathbf{X}_r(i)$ the matrix $\mathbf{X}_r(i) = \frac{\mu}{N} \mathbf{x}_r(i) \mathbf{x}_r^T(i)$, equation (10) takes the form $\mathbf{A}(i) = \mathbf{I} - \mathbf{X}_r(i)$. Let us consider the characteristic polynomial of matrix $\mathbf{A}(i)$:

$$k(\lambda) = \det[\mathbf{A}(i) - \lambda \mathbf{I}] = \det[(1 - \lambda) \mathbf{I} - \mathbf{X}_r(i)] = 0 \quad (13)$$

where λ_l , $l = 1, \dots, M$ are the eigenvalues of matrix $\mathbf{A}(i)$. The characteristic polynomial of matrix $\mathbf{X}_r(i)$ is:

$$k(m) = \det[m \mathbf{I} - \mathbf{X}_r(i)] = 0 \quad (14)$$

where m_i are the eigenvalues of matrix $\mathbf{X}_r(i)$. From the equations (13) and (14) we can see that the relation between the eigenvalues of matrix $\mathbf{A}(i)$ and the eigenvalues of matrix $\mathbf{X}_r(i)$ is:

$$\lambda_l = 1 - m_l, \quad l = 1 \dots M \quad (15)$$

Matrix $\frac{\mu}{N} \mathbf{x}_r(i) \mathbf{x}_r^T(i)$ has rank 1, therefore its coefficients have the form:

$$m_1 = m_2 = \dots m_{M-1} = 0 \\ m_M = \frac{\mu}{N} \mathbf{x}_r^T(i) \mathbf{x}_r(i) \quad (16)$$

From equation (15) we can see that the eigenvalues of matrix $\mathbf{A}(i)$ have the form:

$$\lambda_1 = \lambda_2 = \dots \lambda_{M-1} = 1 \\ \lambda_M = 1 - \frac{\mu}{N} \mathbf{x}_r^T(i) \mathbf{x}_r(i) \quad (17)$$

In order the coefficients to converge, the step-size μ must be chosen in such a way that all the eigenvalues will lie inside the unit circle. This is true for the eigenvalues $\lambda_1 \dots \lambda_{M-1}$. We have to check the above condition for the eigenvalue $\lambda_M : -1 < \lambda_M < 1$. Thus, the following bound on μ is found, according to (17):

$$0 < \mu < \frac{2N}{\mathbf{x}_r^T(i)\mathbf{x}_r(i)} \quad (18)$$

Therefore we can conclude that the system converges, when the step-size μ satisfies (18).

4. DYNAMIC RANGE

Before we design a Cellular Adaptive L -filter, it is necessary to know the dynamic range of its coefficients in order to guarantee that it will satisfy our assumptions on the dynamical equation stipulated in a preceding section. In order to study the dynamic range we shall use the sum of filter coefficients as a figure of merit. The following theorem provides the foundation for our design.

Theorem : The sum of the coefficients $Sa(i) = \sum_{m=1}^M a_m(i; k)$ in a cellular adaptive LMS L -filter is bounded for all $k > 0$ and an upper bound can be computed by the following formula:

$$Sa_{max}(i) = \mu S_{max} \sum_{m=1}^M x_{(m)}(i) \quad (19)$$

where S_{max} is the maximum input value and $x_{(m)}(i)$ is the $m = 1, \dots, M$ element of the ordered-tap input vector $\mathbf{x}_r(i)$. *Proof :* Let us consider the matrix $\mathbf{A}(i)$. For step-size $\mu = \frac{2N}{\mathbf{x}_r^T(i)\mathbf{x}_r(i)}$, which is within the theoretical bounds (18), matrix $\mathbf{A}(i)$ can be considered as an idempotent matrix (i.e., $\mathbf{A}^2(i) = \mathbf{A}(i)$). This feature of matrix $\mathbf{A}(i)$ results the maximum element of the vector $\mathbf{a}(i; k)$ to depend only on the maximum element of the last part of equation (9). However, the following equality is valid:

$$a_m(i; k) = [\mathbf{g}(i) - \mathbf{f}(i; k-1)]_m = \frac{\mu}{N} x_{(m)}(i) \left(\sum_{j \in \mathcal{N}(i)} s(j) - \sum_{j \in \mathcal{N}^*(i)} \mathbf{x}_r^T(i) \mathbf{a}(j; k-1) \right) \quad (20)$$

where $a_m(i; k)$ and $[\mathbf{g}(i) - \mathbf{f}(i; k-1)]_m$ is the $m = 1, \dots, M$ element of the vectors $\mathbf{a}(i; k)$ and $[\mathbf{g}(i) - \mathbf{f}(i; k-1)]$, respectively. The sum of $s(j)$ is bounded as follows:

$$\sum_{j \in \mathcal{N}(i)} s(j) \leq N S_{max} \quad (21)$$

where S_{max} is the maximum input value. If X_{min} is the minimum noisy pixel value, the following inequality results:

$$\begin{aligned} \sum_{j \in \mathcal{N}^*(i)} \mathbf{x}_r^T(j) \mathbf{a}(j; k-1) &= \sum_{j \in \mathcal{N}^*(i)} \sum_{n=1}^M x_{(n)}(j) a_n(j; k-1) \\ &\geq X_{min} \sum_{n=1}^M a_n(j; k-1) \end{aligned} \quad (22)$$

We can assume that the sum of the initial coefficients is 1 (e.g. in the case when the initial coefficients are equal to the

coefficients of the arithmetic mean). This assumption leads to the condition $\sum_{m=1}^M a_m(i; 0) = 1, \forall i$. Using the equations (21) and (22) and the above condition, the equation (9) takes the form:

$$\begin{aligned} \sum_{m=1}^M a_m(i; k) &\leq \mu S_{max} \sum_{m=1}^M x_{(m)}(i) \\ &- \sum_{z=1}^{k-1} \left(-\frac{\mu}{N}\right)^{l+1} N(N-1)^l X_{min}^l S_{max} \left(\sum_{m=1}^M x_{(m)}(i) \right)^{l+1} \\ &+ \left(-\frac{\mu}{N}\right)^k (N-1)^k X_{min}^k \left(\sum_{m=1}^M x_{(m)}(i) \right)^k \end{aligned} \quad (23)$$

The step-size μ takes values within the range $[0, 1]$ and it is normally several magnitude less than 1. Thus $\mu^k \ll \mu, \forall k > 1$. Therefore the third part of (23) is much smaller than the second one. In particular, in the cases where $X_{min} = 0$, the third part of (23) becomes zero. From the above we can consider that the equation (23) can take the form :

$$\sum_{m=1}^M a_m(i; k) \leq \mu S_{max} \sum_{m=1}^M x_{(m)}(i) \quad (24)$$

Now let :

$$Sa_{max}(i) = \mu S_{max} \sum_{m=1}^M x_{(m)}(i) \quad (25)$$

Since $Sa_{max}(i)$ is independent of the iteration k and the cell $C(i)$ for all k and i , the bound on the states of the cells $\sum_{m=1}^M a_m(i; k)$ is finite and can be computed via formula (19).

5. SIMULATION RESULTS

We shall demonstrate the performance of the proposed Cellular Adaptive LMS L -filters through two sets of experiments by employing the noise reduction index NR, defined as the ratio of the output noise power to the input noise power, i.e.:

$$NR = 10 \log \frac{\sum_{i=1}^N \sum_{j=1}^N (y(i, j) - d(i, j))^2}{\sum_{i=1}^N \sum_{j=1}^N (x(i, j) - d(i, j))^2} \quad (26)$$

In (26) $d(i, j)$ is the original image pixel, $x(i, j)$ denotes the value of the same image pixel corrupted by noise and $y(i, j)$ is the filter output at the same image pixel. The images have been corrupted by mixed additive Gaussian noise having zero mean and standard deviation 10 and impulsive noise with probability $p = 10\%$ (both negative and positive impulses with equal probability). A cellular network of 256×256 processing cells has been used. Each processing cell was directly connected to its 2 nearest horizontal neighboring cells. The procedure we have employed in our experiments is as follows. Initially, we find the optimal coefficients of the classical L -filters [5]-[6]. In the next step, that we call iteration number one (i.e., $k = 1$), the adaptation of the cellular adaptive L -filter is initialized with the optimal coefficients that we got in the previous step. In the

next iteration, the initial coefficients of the cellular adaptive L -filter are the coefficients that we got in the end of the previous iteration, and so on.

In the first set of experiments we have examined the performance of the cellular adaptive L -filters in the case of still images. We have filtered the second frame of the sequence with the proposed cellular adaptive filters for a number of iterations. The experimentally found optimal value of the step-size is $\mu_0 = 10^{-5}$ and it has been observed that it is within the theoretical bounds (18). The scanning of the image was performed in the prime-row manner. Table 1 shows the NR we obtained at the end of each iteration. We stop at the sixth iteration, because no further performance improvement can be seen. What we can see from Table 1 is that the NR in the case of cellular adaptive L -filters after same iterations converges to an upper limit. If the ordinary adaptive L -filter (3) were used with the same initial conditions a NR of 16.17 dB would be obtained. That is, the cellular adaptive L -filters yield a 1.5 dB higher NR already after the first iteration. It is seen that a 3 dB higher NR is attained at the end of the sixth iteration. Finally, if an arbitrary choice for the initial filter coefficients were made, the same superior performance would have been obtained, but the required number of iterations would be greater.

The same conclusions can be drawn from the figures. In Figure 2a is displaced the original image and in Figure 2b the corrupted image. For comparison reasons, the output of the adaptive L -filter for still images with initial values the optimum is shown in Figure 2c. The best results obtained, i.e. the output of the Cellular adaptive filter in the case of still images, of frame 2, in the sixth iteration is shown in the Figures 2d. The superiority of Cellular adaptive filters is self-evident.

In the second set of experiments we have studied the performance of the proposed cellular L -filters in the case of image sequence filtering. The first four frames of "Trevor White" image sequence have been used. Each frame has been filtered for a number of iterations. Table 2 shows the NR achieved by the cellular adaptive L -filters after having processed each frame for five iterations. In the same Table, the NR achieved by the classical adaptive L -filter (3) is also listed. It is seen that Cellular adaptive L -filters outperform the ordinary adaptive L -filter in all cases.

6. DISCUSSION AND CONCLUSIONS

In this paper, we propose a new class of adaptive LMS L -filters, called Cellular Adaptive LMS L -filters. Unlike the conventional adaptive filters, our cellular adaptive L -filters are amenable to parallel processing. Moreover, the nearest neighbor interactive property makes the proposed filters much more amenable to VLSI implementation. We have proved some theorems concerning the dynamic range of the coefficients and the steady state of cellular adaptive LMS L -filters.

Simulation experiments have been performed to evaluate the performance of the Cellular Adaptive LMS L -filters in comparison with the classical LMS L -filters. It has been found that the "local nature" of the nearest neighbor interconnections, leads to better adaptation. It has, also, been proven that, after a number of iterations the proposed fil-

ter performance reaches an upper bound. The experiments have been conducted in both the cases of still images and image sequences. The proposed cellular adaptive LMS L -filters yield the best results in all the cases.

7. REFERENCES

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Table 1: Noise reduction in dB achieved by the cellular adaptive L -filters in the case of still images.

Iteration	NR
1	17.535
2	18.159
3	18.722
4	19.009
5	19.146
6	19.17

Table 2: Noise reduction in dB achieved by the cellular adaptive L -filters and the ordinary ones in the case of image sequences.

# frame	Cellular	Ordinary
second	19.146	16.17
third	17.508	16.052
fourth	17.19	16.088

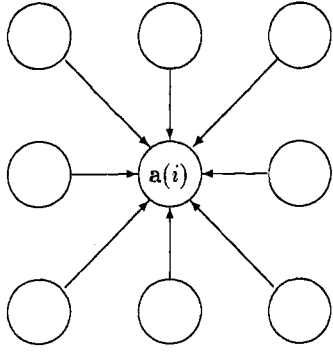
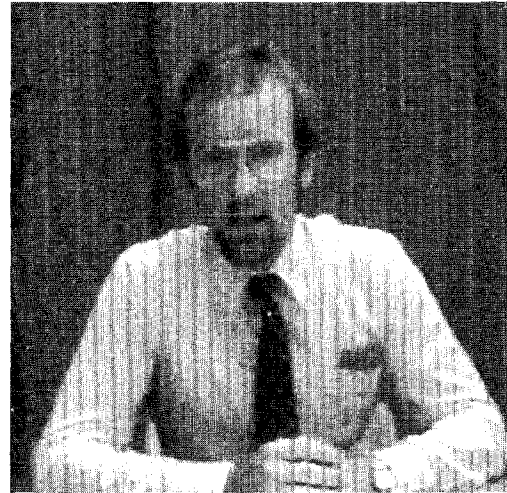
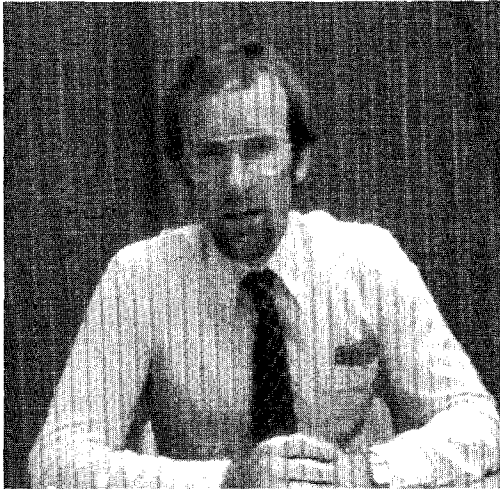


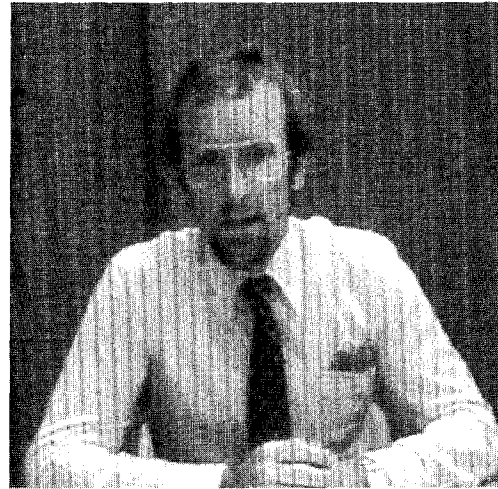
Figure 1 : Interactions within a neighborhood around the cell i



(c)



(a)



(d)



(b)

Figure 2: (a) The original frame number 2 of the image sequence "Trevor White", (b) the corrupted by contaminated Gaussian and impulsive noise, frame 2, (c) the output of the original adaptive L -filter, (d) the output of the cellular adaptive L -filter after the sixth iteration.