EXPLOITING THE SVM CONSTRAINTS IN NMF WITH APPLICATION IN EATING AND DRINKING ACTIVITY RECOGNITION

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ABSTRACT

A novel method is introduced for exploiting the support vector machine constraints in nonnegative matrix factorization. The notion of the proposed method is to find the projection matrix that projects the data to a low-dimensional space so that the data projections between the two classes are separated with maximum margin. Experiments were performed for the task of eating and drinking activity classification. Experimental results showed that the proposed method achieves better classification performance than the state of the art nonnegative matrix factorization and discriminant nonnegative matrix factorization followed by support vector machines classification.

***Index Terms*—** Non-negative Matrix Factorization, Support Vector Machines, Joint Optimization, Maximum Margin Classification

1. INTRODUCTION

Activity recognition is a major research field with broad interest. Apart from the recognition of the most common human activities like walking, running, jumping, bending, sitting and waving, eating and drinking activity recognition consists a research area with a major application field, including monitoring of patients with eating disorders. The implemented eating and drinking activity recognition algorithms either use data obtained from ambient or body-worn sensors, or visual information obtained from one or more cameras. In this paper we present a method that finds application in eating and drinking activity recognition by exploiting only visual information obtained from a single camera. The proposed method exploits the maximum margin constraints of support vector machines (SVMs) [1] in the objective function of nonnegative matrix factorization (NMF) [2]. The intuition behind the proposed framework is to find such a projection matrix that projects the data to a low-dimensional space so that the data projections between the two classes are linearly separable with maximum margin.

Given the nonnegative matrix $\mathbf{X} \in \Re^{N\times M}$, NMF searches for a pair of nonnegative matrices $\mathbf{Z} \in \Re^{N\times L}$, $\mathbf{H} \in \Re^{L\times M}$ whose product approximates $\mathbf{X}$:

$$\mathbf{X} \simeq \mathbf{Z}\mathbf{H}. \tag{1}$$

$\mathbf{X}$ is the data matrix, whose column $\mathbf{x}_j$, $j = 1 \dots M$, represents the j-th element vector of dimension N. $\mathbf{Z}$ is a basis matrix, that projects the data to a space with dimensionality L. By setting $L << N$ data dimensionality reduction is achieved. Finally, $\mathbf{H}$ is the matrix of the data projections. SVMs are employed on the projected data for classification. The objective of SVM is to find the maximum-margin hyperplane, i.e., the hyperplane whose distance from the nearest data of each class is maximal. The elements, whose removal from the training data set change the maximum-margin hyperplane are called support vectors.

Several modifications of NMF and SVM exist, that impose additional constraints to the objective functions of NMF and SVM, respectively, for enhanced discrimination ability of the data projections, such as the discriminant NMF (DNMF) [3] which incorporates the Fisher constraint, the principal components analysis NMF (PCA-NMF) [4], which maximizes the coefficient matrix covariance, the spatially localized NMF (LNMF) [5], which imposes sparseness constraints to the basis matrix and the use of the separable case approximation (SCA) algorithm [6], which computes the SVM on the modified separable training data. In these methods, data representation (NMF) and classification (SVM) occur in independent steps.

In this paper, data representation and classification are formulated into a single objective function, whose optimization aims the projection of the data in a space with reduced dimensions, ensuring that the data projections are separated with maximum margin. More precisely, the maximum-margin hyperplane is defined as a linear combination of the data projections $\mathbf{h}_j$, $j = 1, \dots, M$ and it is incorporated in the objective

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function of NMF.

2. NMF WITH SVM CONSTRAINTS

Let $\mathcal{D} = \{\{\mathbf{x}_j, y_j\}, j = 1, ..., M, \mathbf{x}_j \in \Re^N, y_j \in \{-1, 1\}\}$ be the set of M training data, where $\mathbf{x}_j$ denote the data points and y_j are the corresponding labels. In the standard approach, first NMF is employed in order to find two nonnegative matrices $\mathbf{Z}$, $\mathbf{H}$, that minimize the reconstruction error of the data matrix $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_M]$:

$$\min_{\mathbf{Z},\mathbf{H}} \|\mathbf{X} - \mathbf{Z}\mathbf{H}\|_2 \tag{2}$$

$$\text{s.t. } z_{il} \geq 0, h_{lj} \geq 0, \tag{3}$$

where $\|\cdot\|_2$ denotes the Frobenius norm, or equivalently,

$$\min_{z_{ij}, h_{lj}} \left\{ x_{ij} \ln \left(\frac{x_{ij}}{\sum_{l=1}^{L} z_{il} h_{lj}} \right) + \sum_{l=1}^{L} z_{il} h_{lj} - x_{ij} \right\} \tag{4}$$

$$\text{s.t. } z_{il} \geq 0, h_{lj} \geq 0, \tag{5}$$

where (4) represents the Kullback-Leibler divergence between $\mathbf{X}$ and $\mathbf{ZH}$. Then, SVM is performed on the projected data $\mathbf{h}_j = \mathbf{Z}^T \mathbf{x}_j$ in order to find the hyperplane that maximizes the margin $\frac{2}{\|\mathbf{w}\|}$ between the two classes or, equivalently,

$$\arg\min_{\mathbf{w}} \frac{1}{2} \|\mathbf{w}\|_2^2 \tag{6}$$

$$\text{s.t. } y_j(\mathbf{w}^T \mathbf{h}_j + b) - 1 \geq 0, \forall j = 1, \ldots, M, \tag{7}$$

where $\mathbf{w}$ is the normal vector to the hyperplane and b is the bias. Taking into account the Lagrangian multipliers method and the KKT conditions, the objective of SVM can be written in the form:

$$\min_{a_j} \left\{ \frac{1}{2} \sum_{j=1}^{M} \sum_{k=1}^{M} a_j a_k y_j y_k \mathbf{h}_j^T \mathbf{h}_k - \sum_{j=1}^{M} a_j \right\} \tag{8}$$

$$\text{s.t. } a_j \geq 0, \forall j = 1, \ldots, M, \tag{9}$$

where a_j are the Lagrange multipliers.

In this paper, we exploit the SVM constraints in the optimization framework of NMF, i.e., we want to find a nonnegative base matrix $\mathbf{Z}$ so that, the data projections $\mathbf{h}_j$ minimize the reconstruction error (4) and they are separated with maximum margin by the hyperplane $\mathbf{w}$, which, according to the representer theorem [7], lies in the span of the data projections $\mathbf{w} = \sum_{j=1}^{M} a_j y_j \mathbf{h}_j$. This is accomplished by combining the cost functions (4) and (8) into a single objective function:

$$F(z_{il}, h_{lj}, a_j) = \lambda \sum_{i,j}^{N,M} \left[x_{ij} \ln \left(\frac{x_{ij}}{\sum_l z_{il} h_{lj}} \right) + \sum_l z_{il} h_{lj} - x_{ij} \right]$$

$$+ \frac{1}{2} \sum_{jk}^{M} a_k a_j y_k y_j \sum_{l}^{L} h_{lj} h_{lk} - \sum_{j}^{M} a_j, \tag{10}$$

which we want to minimize with respect to z_{il}, h_{lj}, a_j, subject to the constraints:

$$z_{il} \geq 0, \;\; h_{lj} \geq 0, \;\; a_j \geq 0, \;\; \text{and} \;\; \sum_{i=1}^{N} z_{il} = 1, \;\; \forall l = 1, \ldots, L. \tag{11}$$

The direct minimization of (10) is infeasible. However, a local minimum of (10) can be found, by performing the EM algorithm, since the objective function (10), subject to the constraints (11), is convex with respect to either z_{il}, h_{lj} or a_j. This can be proved by showing that:

$$\frac{\partial^2}{\partial z_{il}^2} F(z_{il}, h_{lj}, a_j)|_{h_{lj}, a_j = \text{constant}} \geq 0 \tag{12}$$

$$\frac{\partial^2}{\partial h_{lj}^2} F(z_{il}, h_{lj}, a_j)|_{z_{il}, a_j = \text{constant}} \geq 0 \tag{13}$$

$$\frac{\partial^2}{\partial a_j^2} F(z_{il}, h_{lj}, a_j)|_{z_{il}, h_{lj} = \text{constant}} \geq 0 \quad . \tag{14}$$

For simplicity in notation, we define:

$$F(z_{il}) = F(z_{il}, h_{lj}, a_j)|_{h_{lj}, a_j = \text{constant}} \tag{15}$$

$$F(h_{lj}) = F(z_{il}, h_{lj}, a_j)|_{z_{il}, a_j = \text{constant}} \tag{16}$$

$$F(a_j) = F(z_{il}, h_{lj}, a_j)|_{z_{il}, h_{lj} = \text{constant}}. \tag{17}$$

Therefore, a local minimum of (10) can be found by minimizing three auxiliary functions $G(z_{il}, z_{il}^{(t)})$, $G(h_{lj}, h_{lj}^{(t)})$ and $G(a_j, a_j^{(t)})$ for the functions $F(z_{il})$, $F(h_{lj})$ and $F(a_j)$, respectively. The function $G(x, x^{(t)})$ is defined to be an auxiliary function for $F(x)$ if $G(x, x^{(t)}) \geq F(x)$ and $G(x, x) = F(x)$. It is proven in [8] that if $G(x, x^{(t)})$ is an auxiliary function for $F(x)$, then the minimization of $G(x, x^{(t)})$ with respect to x leads to minimization of $F(x)$. As a consequence, $F(x)$ is monotonically decreasing under the update rule:

$$x^{(t+1)} = \arg\min_{x} \{G(x, x^{(t)})\}. \tag{18}$$

2.1. Minimization of $F(z_{il}, h_{lj}, a_j)$ w.r.t. z_{il}

The function

$$G(z_{il}, z_{il}^{(t)}) = \lambda \left[\sum_{ij} (x_{ij} \ln x_{ij} - x_{ij}) \right.$$

$$- \sum_{ijl} x_{ij} \frac{z_{il}^{(t)} h_{lj}}{\sum_m z_{im}^{(t)} h_{mj}} \left(\ln z_{il} h_{lj} - \ln \frac{z_{il}^{(t)} h_{lj}}{\sum_m z_{im}^{(t)} h_{mj}} \right)$$

$$\left. + \sum_{ijl} z_{il} h_{lj} \right] + \frac{1}{2} \sum_{jk}^{M} a_k a_j y_k y_j \sum_{l}^{M} h_{lj} h_{lk} - \sum_{j}^{M} a_j \tag{19}$$

is an auxiliary function for the cost function $F(z_{il})$. The derivation of (19) is straightforward from the derivation of the update rule of z_{il} in NMF [9]. The minimization of (19) is performed by setting the partial derivative of $G(z_{il}, z_{il}^{(t)})$ with respect to z_{il} to zero. As a result, $F(z_{il})$ subject to the constraints $z_{il} \geq 0$ and $\sum_{l=1}^{L} z_{il} = 1$ is non-increasing under the following update rules:

$$z_{il}^{'(t+1)} = \sum_j x_{ij} \frac{h_{lj}}{\sum_m z_{im}^{'(t)} h_{mj}} \frac{1}{\sum_j h_{lj}} z_{il}^{'(t)} \quad (20)$$

$$z_{il}^{(t+1)} = \frac{z_{il}^{'(t+1)}}{\sum_{i=1}^{N} z_{il}^{'(t+1)}}. \quad (21)$$

2.2. Minimization of $F(z_{il}, h_{lj}, a_j)$ w.r.t. a_j

The function

$$\begin{aligned} G(a_j, a_j^{(t)}) &= \lambda \sum_{ij} \left[x_{ij} \ln \left(\frac{x_{ij}}{\sum_l z_{il} h_{lj}} + \sum_l z_{il} h_{lj} - x_{ij} \right) \right] \\ &+ \frac{1}{2} \sum_{jk} \frac{A_{jk}^{+} a_k^{(t)}}{a_j^{(t)}} a_j^2 - \frac{1}{2} \sum_{jk} A_{jk}^{-} a_j^{(t)} a_k^{(t)} \\ &\times \left(1 + \ln \frac{a_j a_k}{a_j^{(t)} a_k^{(t)}} \right) - \sum_j a_j, \end{aligned} \quad (22)$$

where $A_{jk} = y_j y_k \sum_l h_{lj} h_{lk}$, $A_{jk}^{+} = \max(A_{jk}, 0)$ and $A_{jk}^{-} = \max(-A_{jk}, 0)$, is an auxiliary function for the cost function $F(a_j)$. The derivation of the auxiliary function (22) is straightforward from the derivation of the update rules of a_j in SVM [10]. By setting the partial derivative of $G(a_j, a_j^{(t)})$ with respect to a_j to zero, the following update rule for a_j is derived:

$$a_j^{(t+1)} = \frac{1 + \sqrt{1 + 4 \sum_k A_{jk}^{+} a_k^{(t)} \sum_k A_{jk}^{-} a_k^{(t)}}}{2 \sum_k A_{jk}^{+} a_k^{(t)}} a_j^{(t)}. \quad (23)$$

2.3. Minimization of $F(z_{il}, h_{lj}, a_j)$ w.r.t. h_{lj}

The function

$$G(h_{lj}, h_{lj}^{(t)}) = G_1(h_{lj}, h_{lj}^{(t)}) + G_2(h_{lj}, h_{lj}^{(t)}), \quad (24)$$

where

$$G_1(h_{lj}, h_{lj}^{(t)}) = \lambda \left[\sum_{ij} (x_{ij} \ln x_{ij} - x_{ij}) - \sum_{ijl} x_{ij} \frac{z_{ij} h_{lj}^{(t)}}{\sum_m z_{im} h_{mj}^{(t)}} \times \left(\ln z_{il} h_{lj} - \ln \frac{z_{il} h_{lj}^{(t)}}{\sum_m z_{im} h_{mj}^{(t)}} \right) + \sum_{ijl} z_{il} h_{lj} \right], \quad (25)$$

$$G_2(h_{lj}, h_{lj}^{(t)}) = \frac{1}{2} \sum_{ljk} \frac{B_{jk}^{+} h_{lk}^{(t)}}{h_{lj}^{(t)}} h_{lj}^2 - \frac{1}{2} \sum_{ljk} B_{jk}^{-} h_{lj}^{(t)} h_{lk}^{(t)} \left(1 + \ln \frac{h_{lj} h_{lk}}{h_{lj}^{(t)} h_{lk}^{(t)}} \right) - \sum_j a_j, \quad (26)$$

$B_{jk} = a_j a_k y_j y_k$, $B_{jk}^{+} = \max(B_{jk}, 0)$ and $B_{jk}^{-} = \max(-B_{jk}, 0)$ is an auxiliary function for the cost function $F(h_{lj})$. The derivation of the auxiliary function (24)-(26) is straightforward from the derivation of the update rules of h_{lj} in NMF [9] and a_j in SVM [10]. By setting the partial derivative of $G(h_{lj}, h_{lj}^{(t)})$ with respect to h_{lj} to zero, the following update rule for h_{lj} is derived:

$$h_{lj}^{(t+1)} = \frac{-\lambda + \sqrt{\lambda + 4 \sum_k B_{jk}^{+} h_{lk}^{(t)} \left(\lambda \sum_i x_{ij} \frac{z_{il}}{\sum_m z_{im} h_{mj}^{(t)}} + \sum_k B_{jk}^{-} h_{lk}^{(t)} \right)}}{2 \sum_k B_{jk}^{+} h_{lk}^{(t)}} h_{lj}^{(t)} \quad (27)$$

2.4. Minimization of $F(z_{il}, h_{lj}, a_j)$ w.r.t. z_{il}, h_{lj} and a_j

Based on the analysis in subsections 2.1-2.3, the cost function $F(z_{il}, h_{lj}, a_j)$ (10) is non-increasing under the iterative update rules (20), (21), (23) and (27). In each iteration, the update rules are computed sequentially, until the cost function converges to a minimum, i.e, when the change in the value of $F(z_{il}, h_{lj}, a_j)$ drops under a threshold. Experimental results showed that the convergence of $F(z_{il}, h_{lj}, a_j)$ occurs in approximately 1000 iterations. In equation (10), λ is a factor that regulates the significance of the NMF part in the objective function. During the first iterations λ takes large values, increasing the significance of the correct data representation. λ decreases exponentially with the number of iterations t, according to $\lambda_0/(1+e)^t$, where the parameter $e << 1$ regulates the decrease rate. λ plays an important role in the classification decision. Experimental results showed that typical values for λ_0 are $\lambda_0 = 100$ or $\lambda_0 = 1000$, while the decrease rate e takes values in the range from 10^{-3} to 10^{-2}.

When the algorithm converges, the train data are projected to the reduced dimensional space using the transpose base matrix $\mathbf{Z}^T$. Alternatively, the data projections $\mathbf{h}_j$ can be estimated using the pseudo-inverse $\mathbf{Z}^{\dagger} = (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T$, or by the multiplicative update rule (27). The maximum margin hyperplane of SVM is computed by:

$$\mathbf{w} = \sum_{j=1}^{M} a_j y_j \mathbf{h}_j \quad (28)$$

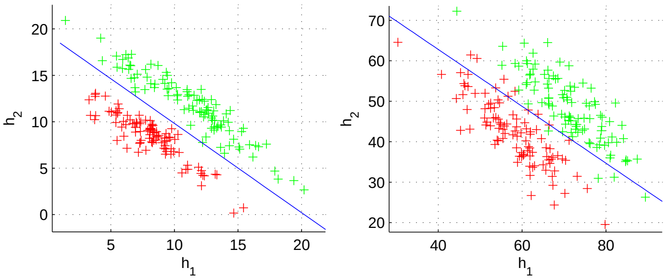
$$b = \frac{1}{|\mathcal{M}_{\mathcal{SV}}|} \sum_{j \in \mathcal{M}_{\mathcal{SV}}} \left(\mathbf{w}^T \mathbf{h}_j - y_j \right) \quad (29)$$

where $\mathcal{M}_{\mathcal{SV}}$ denotes the set of support vectors and, finally, the classification decision is taken according to

$$y_j = \text{sign} \left(\mathbf{w}^T \mathbf{h}_j + b \right). \quad (30)$$

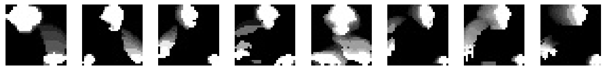
3. EXPERIMENTAL RESULTS

In this section, an experimental evaluation of the proposed NMF with SVM constraints optimization framework is presented. First, the influence of the imposition of the SVM constraints to the cost function of NMF is examined on toy data.



(a) (b)

Fig. 1. Data projections of (a) NMF with SVM constraints and (b) NMF followed by SVM



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 2. MHIs of videos depicting eating with (a) spoon, (b) cutlery, (c) fork, (d) one hand, (e) two hands, and drinking from (f) cup, (g) glass, (h) straw.

Table 1. Classification accuracy (%) of NMF followed by SVM, DNMF followed by SVM and NMF with SVM constraints, algorithms for the AIIA/MOBISERV Database

NMFthenSVM	DNMFthenSVM	NMFwithSVM
78.32%	64.60%	**79.42%**

Then, the classification performance of the proposed method on the AIIA/MOBISERV database for eating and drinking activity recognition is presented.

3.1. Toy Data

The influence of the SVM constraints to the objective function of NMF is examined in a set of toy data that lie in the 10-dimensional space. The toy data belong to two overlapping classes that have Gaussian distributions with mean vectors $\mathbf{m}_1 = 10\mathbf{1}_{10}$ and $\mathbf{m}_2 = 12\mathbf{1}_{10}$, respectively, where $\mathbf{1}_{10}$ denotes the 10-dimensional vector of ones, and covariance matrices $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2 = 1.5\mathbf{I}_{10}$, where $\mathbf{I}_{10} \in \Re^{10\times 10}$ denotes the identity matrix. For each class 100 samples were randomly generated. For visualization reasons, the samples are projected to the two-dimensional space. The scatter plot of the data projections $\mathbf{H}$ after 2000 iterations of the update rules (20), (21), (23) and (27), as well as the maximum-margin hyperplane (28)-(29) are depicted in Figure 1a, while Figure 1b depicts the data projections and the maximum-margin hyperplane produced by the state-of-the-art NMF followed by SVM formulation. From comparison of Figures 1a and 1b we notice that, the imposition of the SVM constraints to the objective function of NMF enforced the linear separation of the projected data. The classification accuracy of the proposed NMF with SVM constraints method is 100%, while the classification accuracy of the state-of-the-art NMF followed by SVM method is 94%.

3.2. AIIA/MOBISERV database

The performance of the proposed NMF with SVM constraints algorithm was tested in the task of eating and drinking activity recognition. Eating and drinking recognition finds application in nutrition assistance systems for frail groups, such as elderly population in the early stage of dementia. The experiments were conducted in the AIIA/MOBISERV eating and drinking activity recognition database[1]. The database contains videos of 12 persons, 6 males and 6 females, with different facial characteristics (eyeglasses, beard, etc.). Each person was recorded in four meal sessions that took place in four different days. Each meal session depicts a person eating with a spoon, cutlery, a fork, one and both hands and drinking from a cup, a glass and a straw. Skin color segmentation was performed in each video, creating binary masks [11]. Finally, the Motion History Images (MHI) [12] (Figure 2) of each activity were extracted and down-scaled to 32×32 pixels. In total, 3969 MHIs where created.

The algorithm performance was tested by using the MHIs as the video features and the leave-one-day-out cross validation method, i.e. 25% of the samples were used for testing. The classification accuracy of the proposed method was compared to the performance of the state-of-the-art NMF, as well as the discriminant NMF (DNMF) [3], which is a popular variant of NMF, followed by SVM classification. The classification accuracies are shown in Table 1. We notice that, the proposed method achieves the highest classification accuracy of 79.42%, followed by the standard NMF followed by SVM classification (78.32%). The classification performance of DNMF followed by SVM has the lowest accuracy of only 64.60%, since it exploits sparse information for data representation that is not discriminative for the specific task.

4. CONCLUSIONS

In this paper a novel method was introduced that finds a nonnegative projection matrix which projects the data to a low-dimensional space, so that the data projections between the two classes are linearly separable with maximum margin. Experimental results on toy data and eating/drinking activity recognition showed the supremacy of the proposed method with respect to the state of the art NMF and DNMF followed by SVM classification.

[1] `http://www.aiia.csd.auth.gr/MOBISERV-AIIA/index.html`

5. REFERENCES

[1] Christopher J. C. Burges, "A tutorial on support vector machines for pattern recognition," *Data Min. Knowl. Discov.*, vol. 2, pp. 121–167, June 1998.

[2] Daniel D. Lee and H. Sebastian Seung, "Learning the parts of objects by non-negative matrix factorization," *Nature*, vol. 401, no. 6755, pp. 788–791, Oct. 1999.

[3] S. Zafeiriou, A. Tefas, I. Buciu, and I. Pitas, "Exploiting discriminant information in nonnegative matrix factorization with application to frontal face verification," *IEEE Transactions on Neural Networks*, vol. 17, no. 3, pp. 683 –695, may 2006.

[4] Yuan Wang, Yunde Jia, Changbo Hu, and Matthew Turk, "Non-negative matrix factorization framework for face recognition," *International Journal of Pattern Recognition and Artificial Intelligence*, vol. 19, pp. 495–511, 2005.

[5] S.Z. Li, Xin Wen Hou, Hong Jiang Zhang, and Qian Sheng Cheng, "Learning spatially localized, parts-based representation," in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2001, vol. 1, pp. 207 – 212.

[6] Dries Geebelen, Johan A. K. Suykens, and Joos Vandewalle, "Reducing the number of support vectors of svm classifiers using the smoothed separable case approximation," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 4, pp. 682–688, april 2012.

[7] Thomas Hofmann, Bernhard Schlkopf, and Alexander J. Smola, "Kernel methods in machine learning," *Annals of Statistics*, vol. 36, no. 3, pp. 1171–1220, 2008.

[8] D. D. Lee and H. S. Seung, "Learning the parts of objects by non-negative matrix factorization.," *Nature*, vol. 401, no. 6755, pp. 788–91, 1999.

[9] Daniel D. Lee and H. Sebastian Seung, "Algorithms for Non-negative Matrix Factorization," in *Advances in Neural Information Processing Systems 13*. Apr. 2001, pp. 556–562, MIT Press.

[10] Fei Sha, Yuanqing Lin, Lawrence K. Saul, and Daniel D. Lee, "Multiplicative updates for nonnegative quadratic programming," *Neural Comput.*, vol. 19, pp. 2004–2031, August 2007.

[11] Ermioni Marami, Anastasios Tefas, and Ioannis Pitas, "Nutrition assistance based on skin color segmentation and support vector machines," in *Man-Machine Interactions 2*, vol. 103 of *Advances in Intelligent and Soft Computing*, pp. 179–187. Springer Berlin / Heidelberg, 2011.

[12] J.W. Davis, "Hierarchical motion history images for recognizing human motion," in *IEEE Workshop on Detection and Recognition of Events in Video*, 2001, pp. 39 –46.